FINANCIAL ENGINEERING
KBP FFŪ & KSTP FIS

Credit Risk Management and Modeling

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Office hours: Tuesday 10-12

Literature

Requirement	Title	Author	Year of Publication
Required	Credit Risk Management and Modeling	Witzany, J.	2010, Oeconomica, pp. 214
Optional	Managing Credit Risk – The Great Challenge for Global Financial Markets	Caouette J.B., Altman E.I., Narayan O., Nimmo R.	2008, 2nd Edition, Wiley Finance, pp. 627
Optional	Credit Risk – Pricing, Measurement, and Management	Duffie D., Singleton K.J.	2003, Princeton University Press, pp.396
Optional	Consumer Credit Models: Pricing, Profit, and Portfolios	Thomas L. C.	2009, Oxford University Press, pp. 400
Optional	Credit Derivatives Pricing Models	Schönbucher P.J.	2003, Wiley Finance Series, pp. 375

Course Organization: project (scoring function development), small midterm test (26.3.), final test (14.5. ?)

Content

- Introduction
- Credit Risk Management
  - Credit Risk Organization
  - Trading and Investment Banking
  - Basel II
- Rating and Scoring Systems
  - Rating Validation
  - Analytical Ratings
  - Automated Rating Systems

Content - continued

  - Expected Loss, LGD, and EAD
  - Basel II Requirements
- Portfolio Credit Risk
  - CreditMetrics
  - Credit Risk+
  - Credit Portfolio View
  - KMV Portfolio Manager
  - Basel II Capital requirements

Content - continued

- Credit Derivatives
  - Overview of Basic Credit Derivatives Products
  - Intensity of Default Stochastic Modeling
  - Copula Correlation Models
  - CDS and CDO Valuation

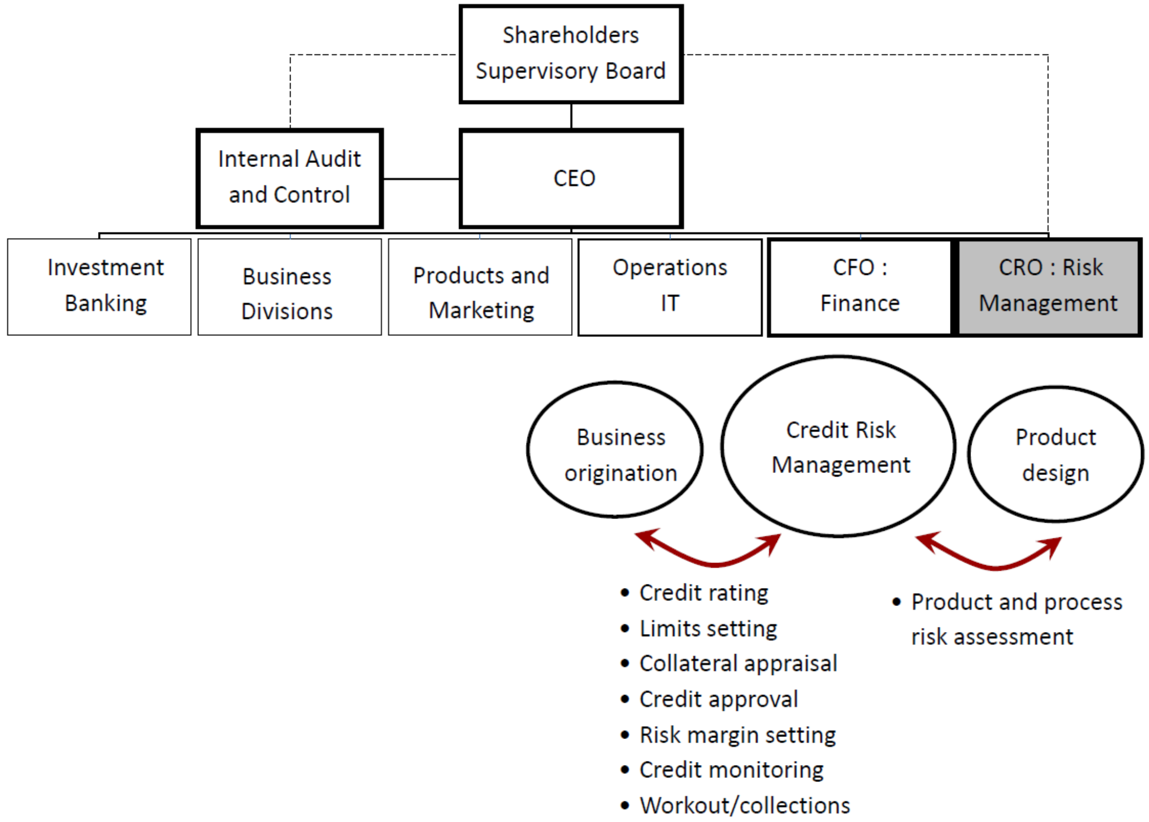
Introduction

- Classical commercial bank credit risk management
  - Corporate loans approval and pricing
  - Retail loans approval and pricing
  - Provisioning and workout
  - Regulatory requirements – Basel II
  - Loan portfolio reporting and management
  - Financial markets (counterparty) credit risk

Introduction

- Credit approval – can we get a crystal ball?
- What is the first, the chicken or the egg, Basel II or credit rating?
- What is new in Basel III?
- Should we start the course with credit derivatives?

Credit Risk Management Organization



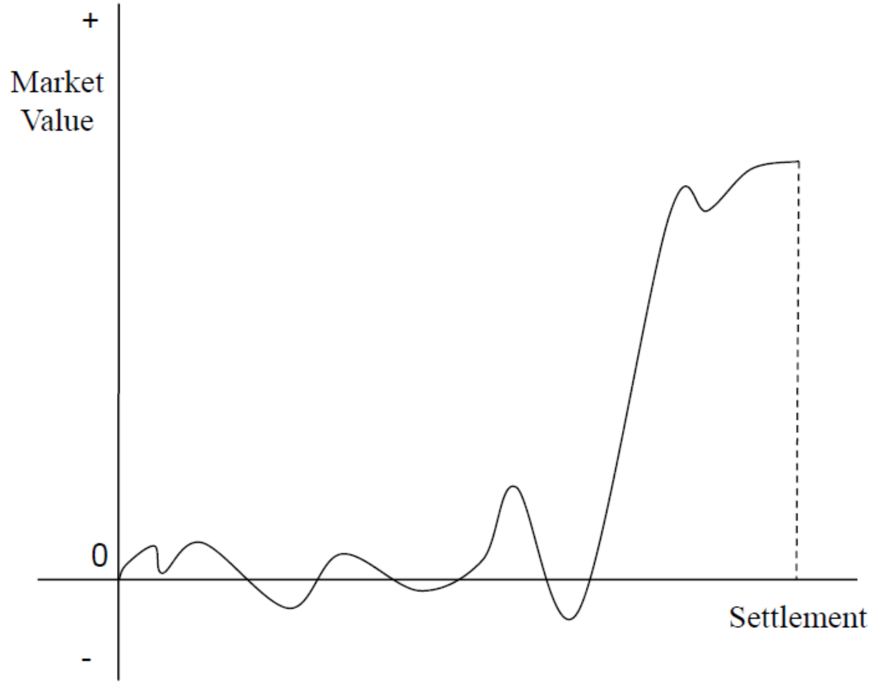
Separation of Powers – Risk Organization Independence!!!

Credit Risk Process

	Credit origination	Monitoring	Workout
Responsibility	• Business units • Credit Risk	• Business units • Credit Risk • Market Risk	• Debt Recovery
Key activities	• Credit rating • Evaluation of applications • Risk adjusted pricing • Collateral valuation	• Monitoring and identification of cases turning bad • Reclassification / Provisioning of bad loans • Corrective actions	• Recovering of bad debts • Closing unsolvable cases

Importance of credit risk information management!!!

Counterparty and Settlement Risk on Financial Markets

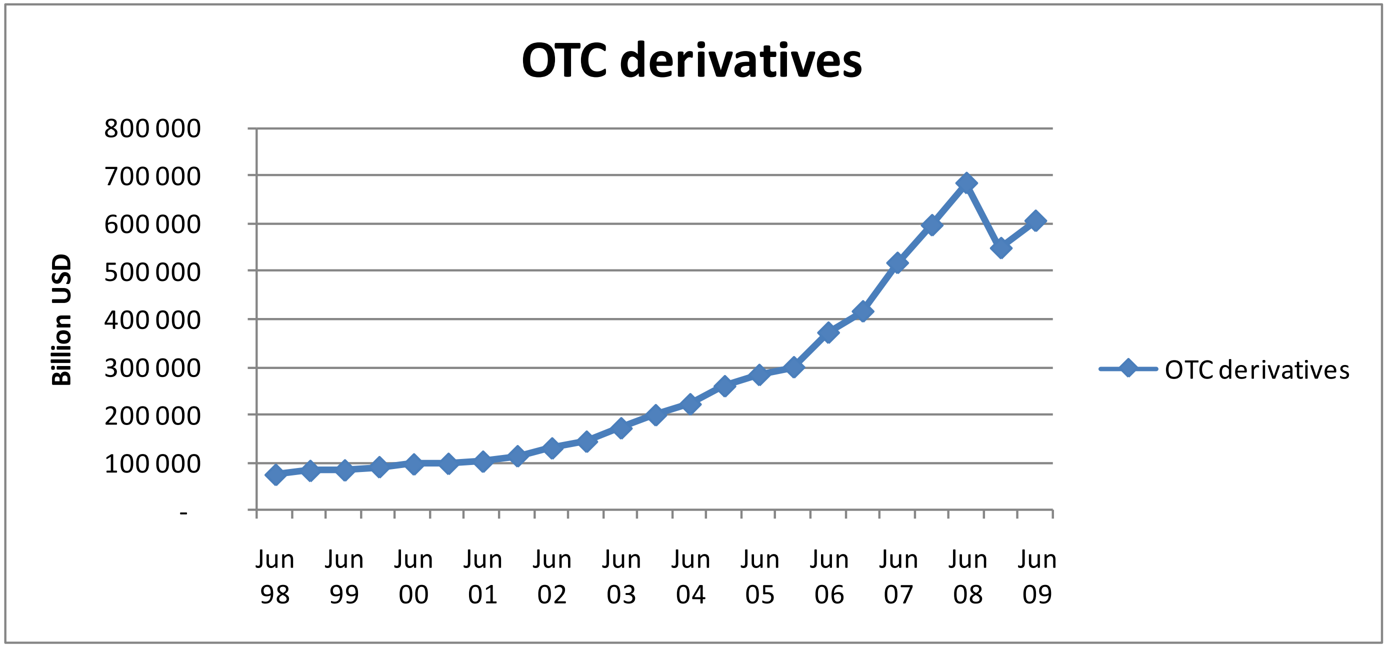


Settlement Risk only at the settlement date
Counterparty Risk over the transaction life

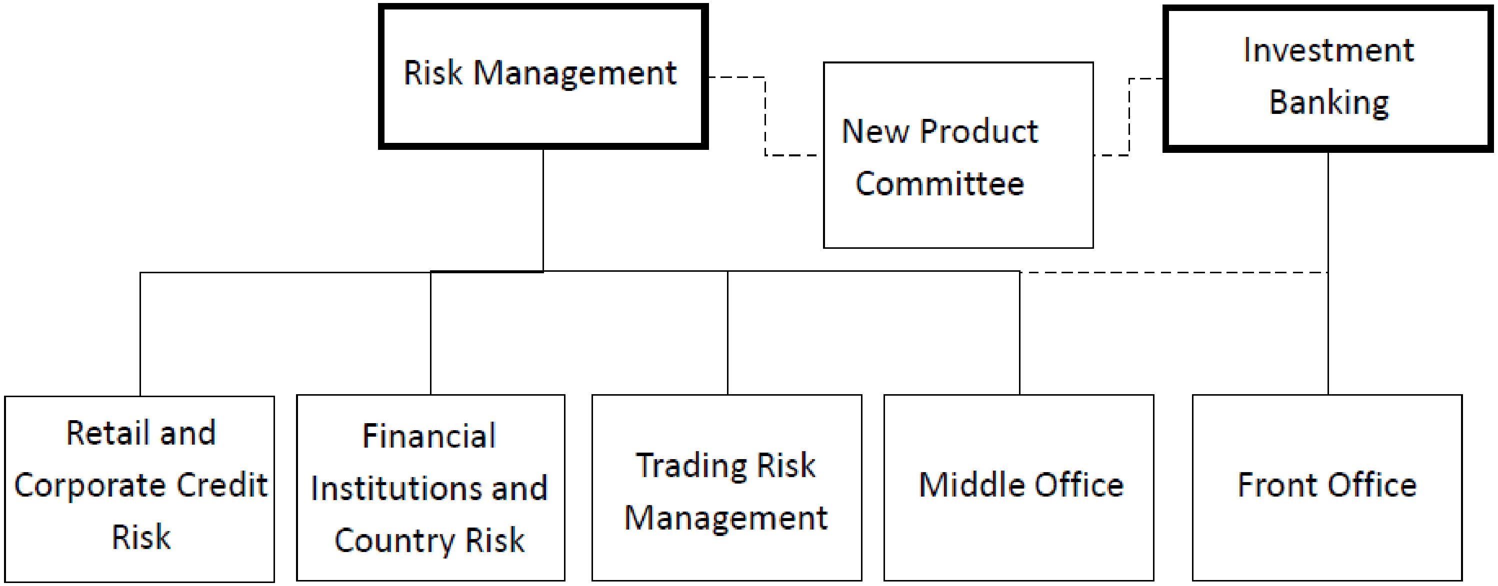
Counterparty Risk Equivalents

$$\text{Exposure} = \max(\text{market value}, 0) + x\% \cdot \text{Nominal amount}$$

Growing global counterparty risk!!!
Could be partially reduced through netting agreements



Trading Credit Risk Organization



Importance of Back Office independence!!!

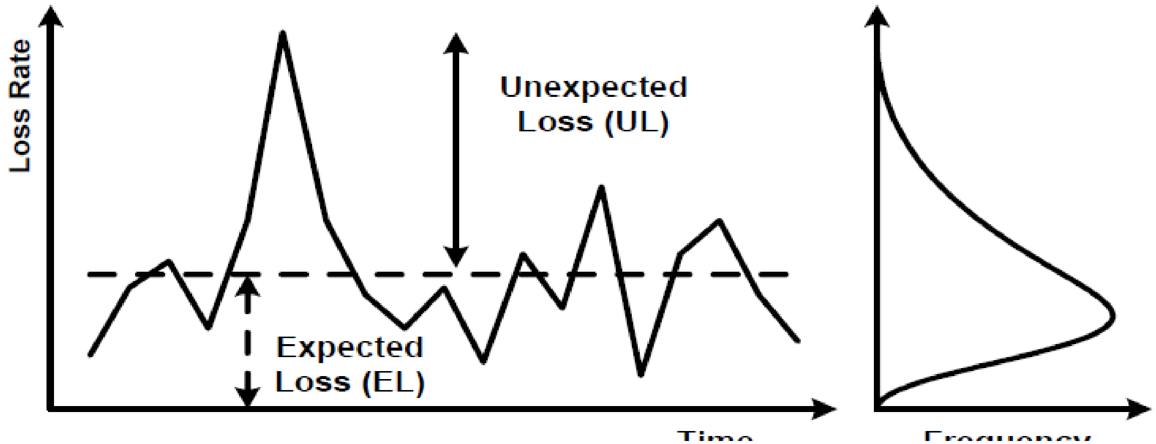
The risks must not be underestimated

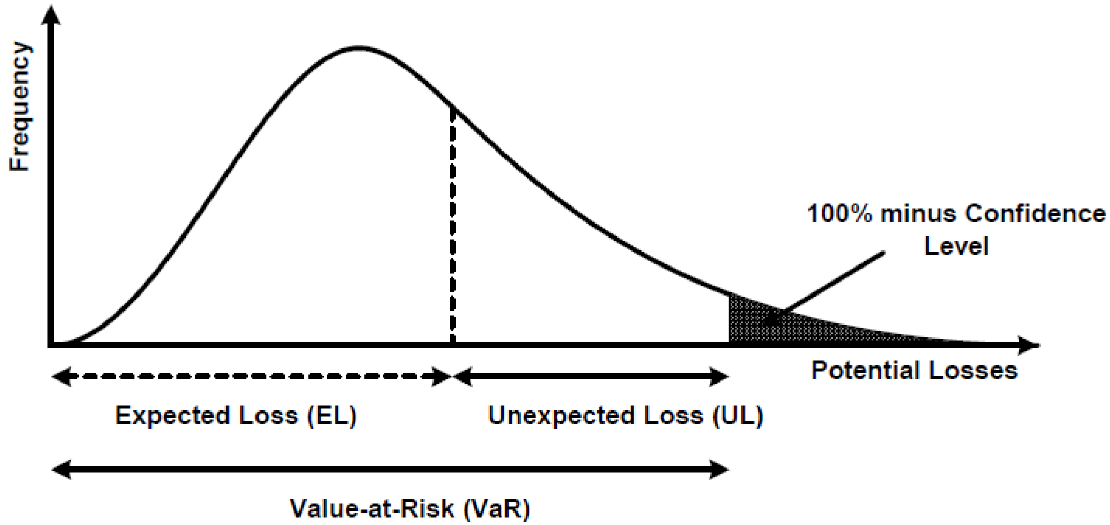
Craig Broderick, responsible for risk management in Goldman Sachs in 2007 said with self-confidence: *“Our risk culture has always been good in my view, but it is stronger than ever today. We have evolved from a firm where you could take a little bit of market risk and no credit risk, to a place that takes quite a bit of market and credit risk in many of our activities. However, we do so with the clear proviso that we will take only the risks that we understand, can control and are being compensated for.”* In spite of that Goldman Sachs suffered huge losses at the end of 2008 and had to be transformed (together with Morgan Stanley) from an investment bank to a bank holding company eligible for help from the Federal Reserve.

Basel II - History

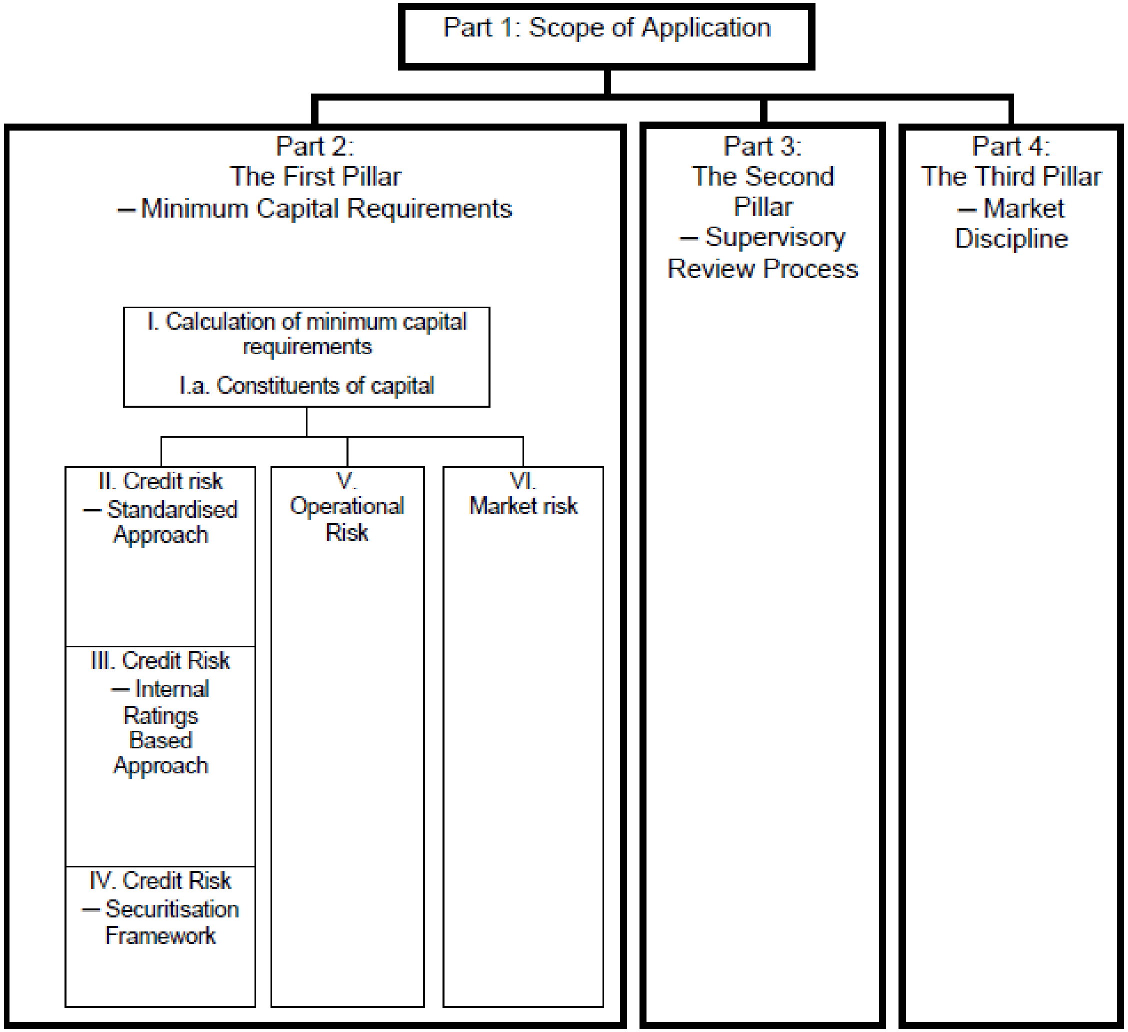
- 1988: 1st Capital Accord – Basel I (BCBS, BIS)
- 1996: Market Risk Ammendment
- 1999: Basel II 1st Consultative Paper
- 2004: The New Capital Accord - Basel II
- 2006: EU Capital Adequacy Directive
- 2007: CNB Provision on Basel II
- 2008: Basel II effective for banks
- 2010: Basel III following the crisis (effective 2013-2019)

Basel II - Principles





Basel II Structure



Basel II Qualitative Requirements

- 441. *Banks must have **independent credit risk control units** that are responsible for the design or selection, implementation and performance of their internal rating systems. The unit(s) must be functionally independent from the personnel and management functions responsible for originating exposures. Areas of responsibility must include:*
- *Testing and monitoring internal grades;*
- *Production and analysis of summary reports from the bank's rating system, to include historical default data sorted by rating at the time of default and one year prior to default, grade migration analyses, and monitoring of trends in key rating criteria;*
- *Implementing procedures to verify that rating definitions are consistently applied across departments and geographic areas;*
- *Reviewing and documenting any changes to the rating process, including the reasons for the changes; and*
- *Reviewing the rating criteria to evaluate if they remain predictive of risk. Changes to the rating process, criteria or individual rating parameters must be documented and retained for supervisors to review.*

Basel II Qualitative Requirements

730. ***The bank's board of directors has responsibility for setting the bank's tolerance for risks.*** *It should also ensure that management establishes a framework for assessing the various risks, develops a system to relate risk to the bank's capital level, and establishes a method for monitoring compliance with internal policies. It is likewise important that the* ***board of directors adopts and supports strong internal controls and written policies*** *and procedures and ensures that management effectively communicates these throughout the organization….*

Content

- ✓ Introduction
- ✓ Credit Risk Management
- ➢ Rating and Scoring Systems
- Portfolio Credit Risk Modeling
- Credit Derivatives

Rating/PD Prediction - Can We Predict the Future?

Present time

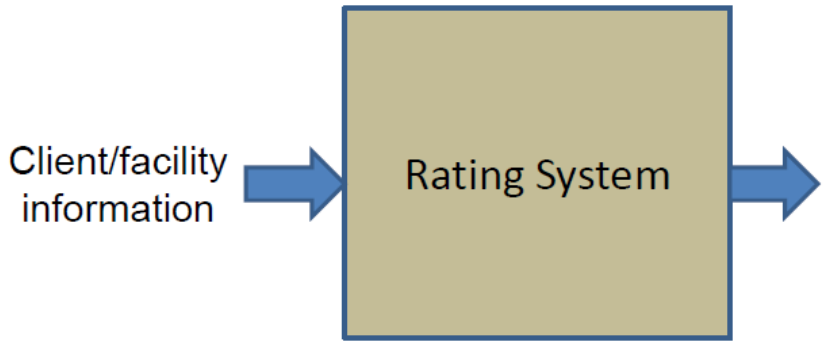
Time horizon?
Default/Loss Definition?

PAST
Historical data
Experience
Know-how

FUTURE?
Probabilty of Default?
Expected Loss?
Loss distribution?

Actual Client/ Exposure/ Application Information

Rating and Scoring Systems



Rating/scoring in a scale

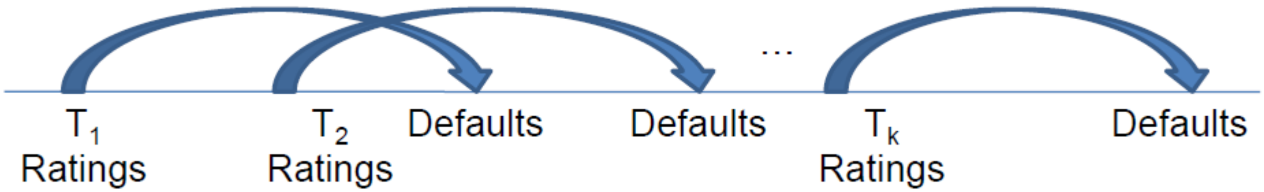
C	...	AAA
C	...	Aaa
1	...	21
1	...	1000

- Rating/scoring system could be in general a black box
- How do we measure quality of a rating system?

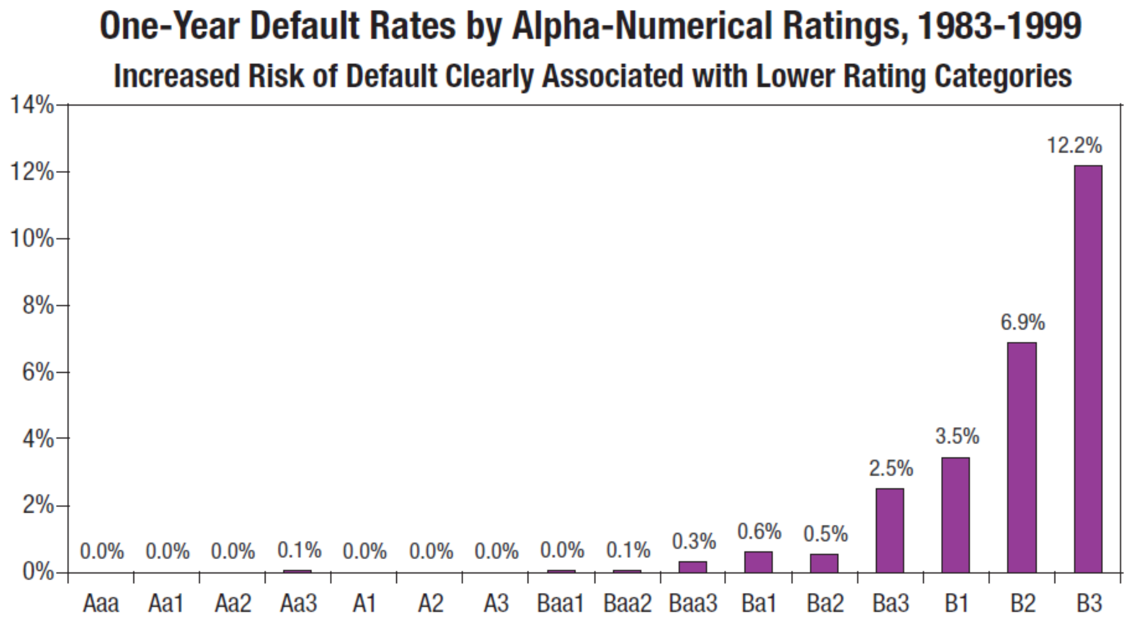
Rating Validation – Performance Measurement

- Validation – measurement of prediction power – does the rating meet our expectations?
- Exact definitions:
  - Time horizon
  - Debtor or facility?
  - Current situation or conditional on a new loan?
  - Definition of default?
  - Probability of default prediction or just discrimination between better and worse?

Default History

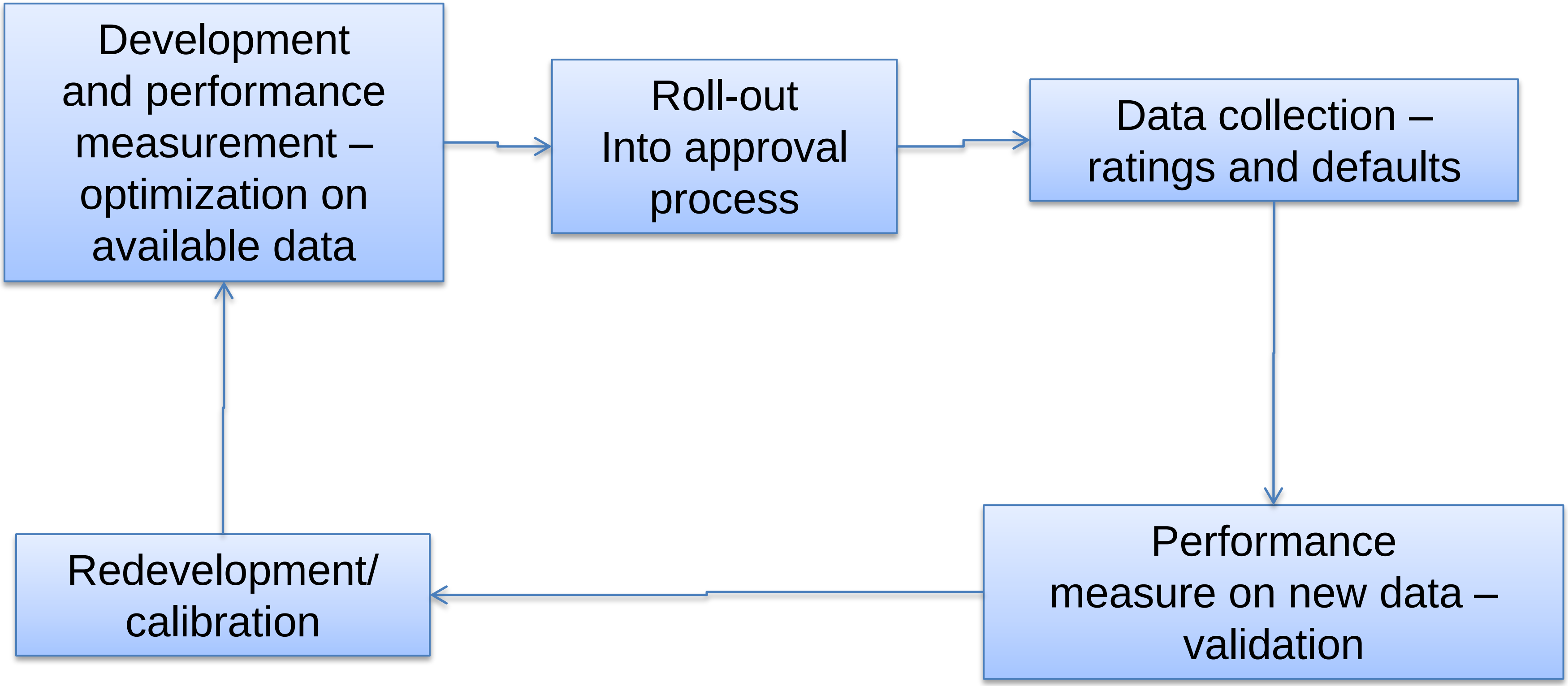


Do we observe less defaults for better grades?



Source: Moody's

Rating System Continuous Development Process



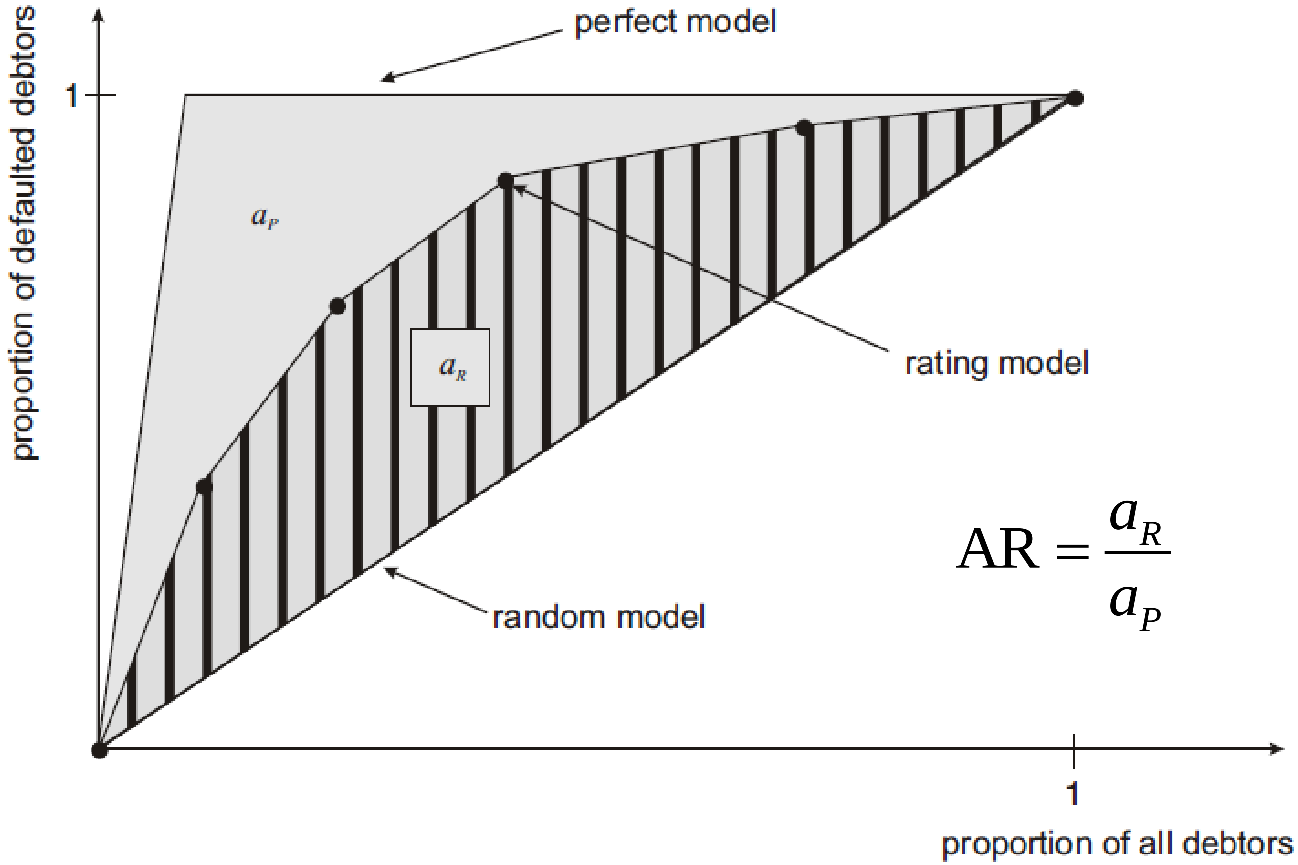
Discriminative Power of Rating Systems

- Let us have a rating system and let us assume that we know the future (good and bad clients)
- Let us have a bad X and a good Y
- Correct signal: rating(X)<rating(Y)
- Wrong signal: rating(Y)<rating(X)
- No signal: rating(X)=rating(Y)
- In a discrete setting we may count the numbers of the signals

Discriminative Power of Rating Systems

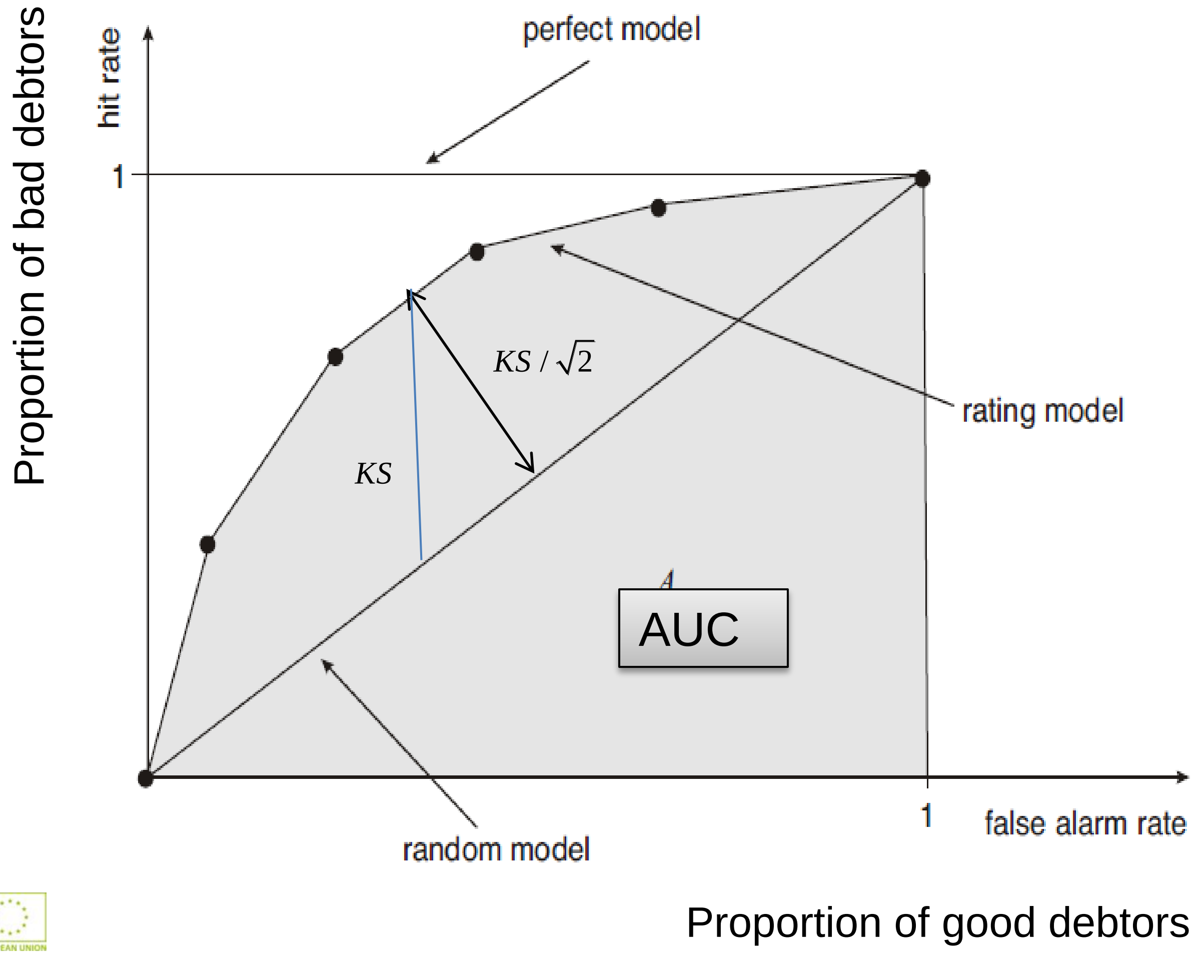
- To measure prediction power let us count/measure probabilities of the signals
- p_1 = Pr[correct signal]
- p_2 = Pr[wrong signal]
- p_3 = Pr[no signal]
- AR = Accuracy Ratio = $p_1 - p_2$
- AUC = Area Under Receiver Operating Characteristic Curve = $p_1 + p_3/2 = (AR+1)/2$

Cumulative Accuracy Profile



$$AR = \frac{a_R}{a_P}$$

Receiver Operating Characteristic Curve



AR, AUC, and KS

- The probabilistic and geometric definitions of AR and AUC are equivalent
- It follows immediately from the probabilistic definitions that AR=2AUC-1
- Related Kolmogorov-Smirnov statistics can be defined as the maximum distance of the ROC curve from the diagonal multiplied by $\sqrt{2}$

Empirical Estimations of AR (and AUC)

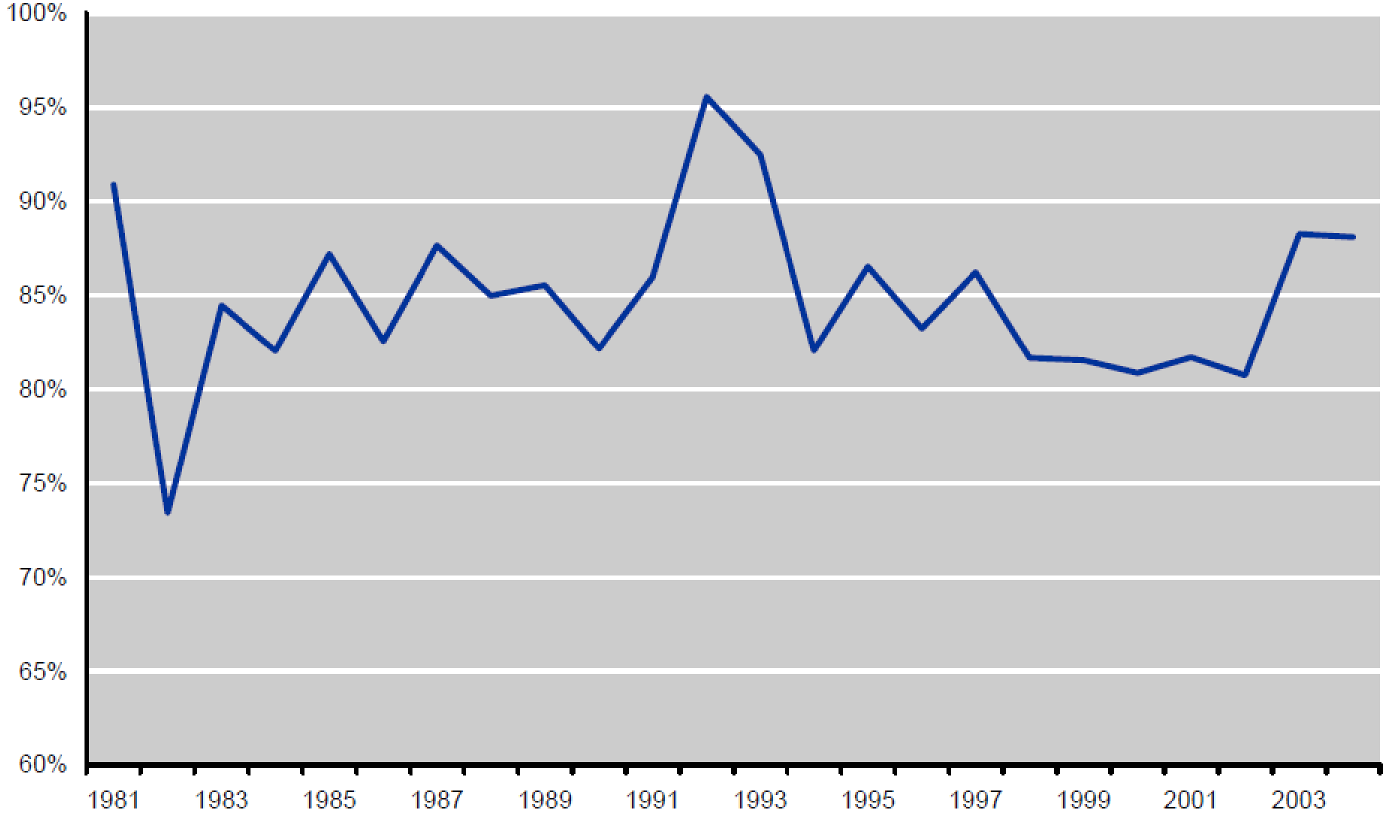
- Empirical AR or AUC depend on the sample used
- Generally there is an error between the theoretical AR and an empirical one
- This must be taken into account comparing the Accuracy ratios of two ratings
- The are however tests of statistical significance

In-sample or Out-sample

- A rating system is usually developed (trained) on a historical sample of defaults
- When the AR of the system is calculated on the training sample (in sample) then we generally must expect better values than on an independent (out sample)
- Real validation should be done on a sample of data collected after the development

S&P Rating Performance

Chart 16. **One-Year Gini Coefficients**



Source: Standard & Poor's Global Fixed Income Research; Standard & Poor's CreditPro® 7.0.

Comparing two rating systems on the same validation sample

	Gini	St.Error	95% Confidence interval	
Rating1	69,00%	1,20%	66,65%	71,35%
Rating2	71,50%	1,30%	68,95%	74,05%

H0: Gini(Rating1)=Gini(Rating2)

$\chi 2(1)=$ 9,86

Prob>$\chi 2(1)=$ 0,17%

We can conclude that the second rating has a better performace than the first on the 99% probability level

Measures of Correctness of Categorical Predictions

- In practice we use a cut-off score in order to decide on acceptance/rejection of new applications
- The goal is to accept good applicants and reject bad applicants
- Or in fact minimize losses caused bad accepted applicants and good rejected applicants (opportunity cost)

Measures of Correctness of Categorical Predictions

	Actual goods	Actual bads	Total
Approved	g_G	g_B	g
Rejected	b_G	b_B	b
Actual numbers	n_G	n_B	n

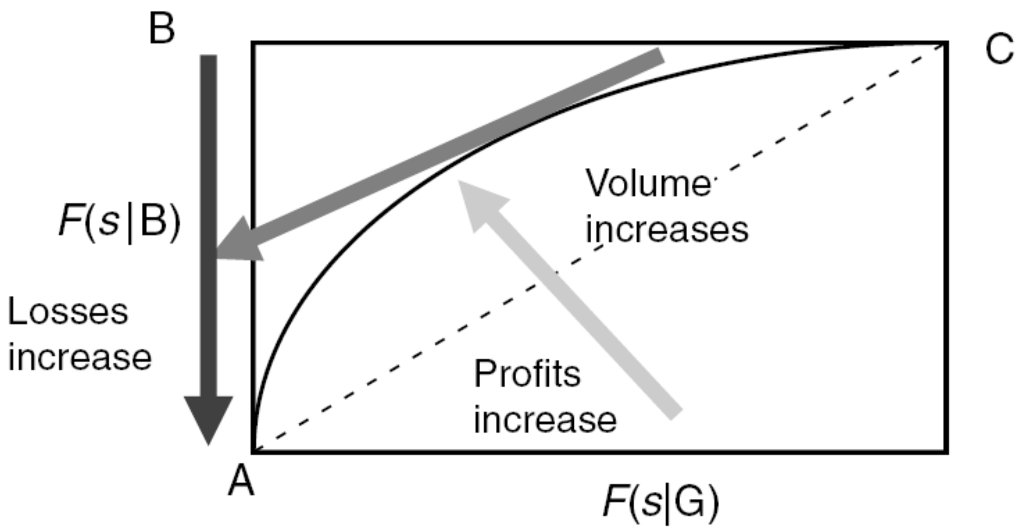
	Actual goods	Actual bads	Total
Approved	$\pi_G F^c(s_c \mid G)$	$\pi_B F^c(s_c \mid B)$	$F^c(s_c \mid G)$
Rejected	$\pi_G F(s_c \mid G)$	$\pi_B F(s_c \mid B)$	$F(s_c \mid G)$
Total actual	π_G	π_B	1

Measures of Correctness of Categorical Predictions

- Generally we need to minimize the weighted cost of errors

$$WCE = l\pi_B F^c(s_c \mid B) + q\pi_G F(s_c \mid G) = q\pi_G \;\; C \cdot F^c(s_c \mid B) + F(s_c \mid G)$$

$$w(s_c) = C \cdot F^c(s_c \mid B) + F(s_c \mid G) = C - C \cdot F(s_c \mid B) + F(s_c \mid G)$$



$$C = \frac{l\pi_B}{q\pi_G}$$

Cut-off Optimization

- **Example:** Scorecard 0…100, proposed cut-off 40 or 50
- Validation sample: 10 000 applications, 7 000 approved where we have information on defaults
- We estimate F(40|G)=12%, F(50|G)=14%, F(40|B)=70%, F(50|B)=76%
- Based on scores assigned to all applications we estimate π_B=10%.
- Calculate the total and weighted rate of errors if l=60% and q=10%. Select the optimal cut-off.

Validation of PD estimates

- If the rating is connected with expected PDs then we ask how close is our experienced rate of default on rating grades to the expected PDs
- **Hosmer-Lemeshow Test** – one comperehensive statistics asymptotically χ^2 with number of rating grades – 2 degrees of freedom

$$S_N^{\chi^2} = \sum_{s=1}^{N} \frac{n_s (PD_s - p_s)^2}{PD_s (1 - PD_s)} = \sum_{s=1}^{N} \frac{(n_s PD_s - d_s)^2}{n_s PD_s (1 - PD_s)}$$

Hosmer-Lemenshow Test Example

rating (s)	PD_s	p_s	N_s	
1	30%	35,0%	50	0,595238095
2	10%	8,0%	150	0,666666667
3	5%	7,0%	70	0,589473684
4	1%	1,5%	50	0,126262626
			Hos.-Lem.	**1,977641072**
			p-value	0,37201522

H_0 is not rejected on 90% probability level
Low p-value would be a problem

Binomial Test

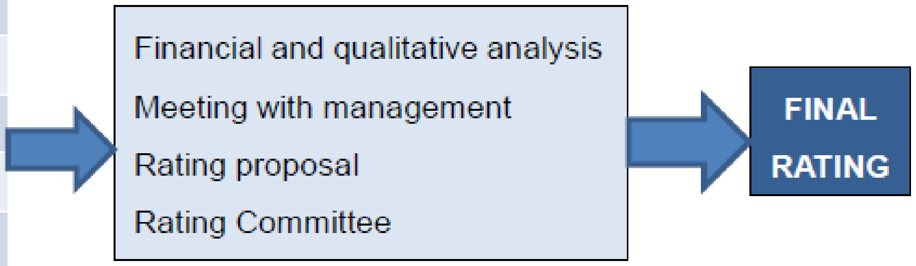
- Only for one grade, one-sided
- If $P \leq PD_s$ then the probability of observing or more defaults is at most

$$\tilde{B}(d; N_s, PD_s) = \sum_{j=d}^{N_s} \binom{N_s}{j} PD_s^{\,j} (1 - PD_s)^{N_s - j}$$

- **Example:** PD=1%, 1000 observations, 13 defaults, is it OK or not?
- The issue of correlation can be solved with a Monte Carlo simulation

Analytical (Expert) Ratings

Business Risk	Financial Risk
Industry Characteristics	Financial Characteristics
Competitive Position	Financial Policy
Marketing	Profitability
Technology	Capital Structure
Efficiency	Cash Flow Protection
Regulation	Financial Flexibility
Management	Business Risk



Standard & Poor's Rating Process

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Key Financial Ratios

Category	Ratio
Operating performance	Return on equity (ROE) = Net income/Equity Return on assets (ROA) = Net income/Assets Operating Margin = EBITDA/Sales Net Profit Margin = Net income/Sales Effective tax rate Sales/Sales last year Productivity = (Sales-Material costs)/Personal costs
Debt service coverage	EBITDA/Interest Capital expenditure/Interest
Financial leverage	Leverage = Assets/Equity Liabilities/Equity Bank Debt/Assets Liabilities/Liabilities last year
Liquidity	Current ratio = Current assets/Current Liabilities Quick ratio = Quick Assets/Current Liabilities Inventory/Sales
Receivables	Aging of receivable (30,60,90,90+ past due) Average collection period

External Agencies' Rating Symbols

Rating	Interpretation
	Investment Grade Ratings
AAA/Aaa	Highest quality; extremely strong; highly unlikely to be affected by foreseeable events.
AA/Aa	Very High quality; capacity for repayments is not significantly vulnerable to foreseeable events.
A/A	Strong payment capacity; more likely to be affected by changes in economic circumstances.
BBB/Ba	Adequate payment capacity; a negative change in environment may affect capacity for repayment.
	Below Investment Grade Ratings
BB/Ba	Considered speculative with possibility of developing credit risks.
B/B	Considered very speculative with significant credit risk.
CCC/Caa	Considered highly speculative with substantial credit risk.
CC/Ca	Maybe in default or wildly speculative.
C/C/D	In bankruptcy or default.

Through the Cycle (TTC) or Point in Time (PIT) Rating

- External agencies do not assign fixed PDs to their ratings
- Observed PDs for the rating classes depend on the cycle
- Should the rating express the actual expected PD conditional on current economic conditions (PIT) or should it be independent on the cycle (TTC)
- External ratings are hybrid - somewhere between TTC and PIT

The Risk of External Rating Agencies

- The rating agencies have become extremely influential – cover $34 trillion of securities
- Ratings are paid by issuers – conflict of interest
- Certain new products (CDO) have become extremely complex to analyze
- Many investors started to rely almost completely on the ratings
- Systematic failure of rating agencies => global financial crisis

Internal Analytical Ratings

- Asset based – asset valuation
- Unsecured general corporate lending or project lending – ability to generate cash flow
- Depends on internal analytical expertise
- Retail, Small Business, and SME also usually depend at least partially on expert decisions

Automated Rating Systems

- Econometric models – discriminant analysis, linear, probit, and logit regressions
- Shadow rating, try to mimic analytical (external) rating systems
- Neural networks
- Rule-based or expert systems
- Structural models, e.g. KMV based on equity prices

Altman's Z-score

- Altman (1968), maximization of the discrimination power by $z = \sum_{i=1}^{k} \beta_i x_i$ on a dataset of 33 bankrupt + 33 non-bankrupt companies

$$Z = 1.2x_1 + 1.4x_2 + 2.3x_3 + 0.6x_4 + 0.999x_5$$

Variable	**Ratio**	**Bankrupt group mean**	**Non-bankrupt group mean**	**F-ratio**
x_1	Working capital/Total assets	-6.1%	41.4%	32.60
x_2	Retained earnings/Total assets	-62.6%	35.5%	58.86
x_3	EBIT/Total assets	-31.8%	15.4%	25.56
x_4	Market value of equity/Book value of liabilities	40.1%	247.7%	33.26
x_5	Sales/Total assets	1.5	1.9	2.84

Altman's Z-score

- Maximization of the discrimination function – maximization between bad/good group variance and minimization within group variance

$$\frac{N_1(\bar{z}_1 - \bar{z})^2 + N_2(\bar{z}_2 - \bar{z})^2}{\sum_{i=1}^{N_1}(z_{1,i} - \bar{z}_1)^2 + \sum_{i=1}^{N_2}(z_{2,i} - \bar{z}_2)^2}$$

- Importance of selection of variables
- In fact the approach is similar to the least squares linear regression $y_i = \boldsymbol{\beta}' \cdot \mathbf{x}_i + u_i$

Logistic Regression

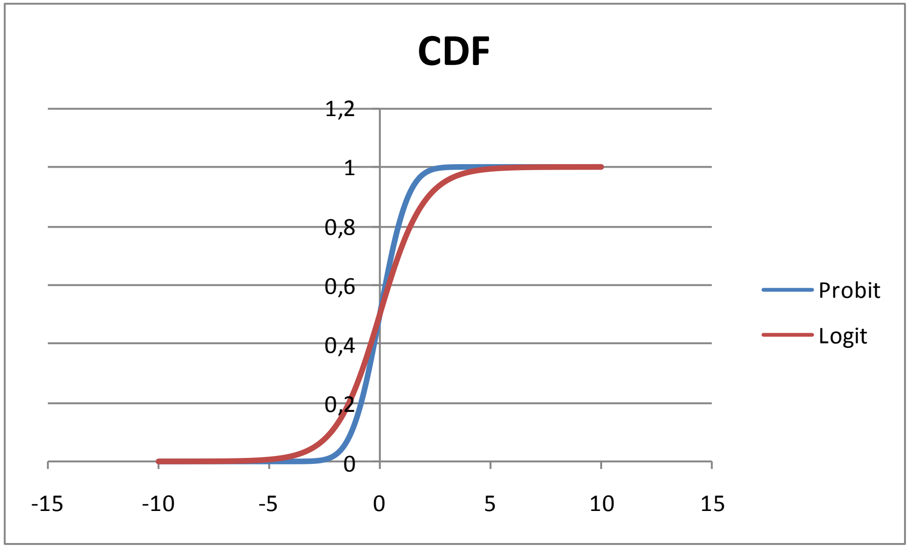
- Latent credit score $y_i^* = \boldsymbol{\beta}' \cdot \mathbf{x}_i + u_i$
- Default iff $y_i^* \le 0$, i.e.

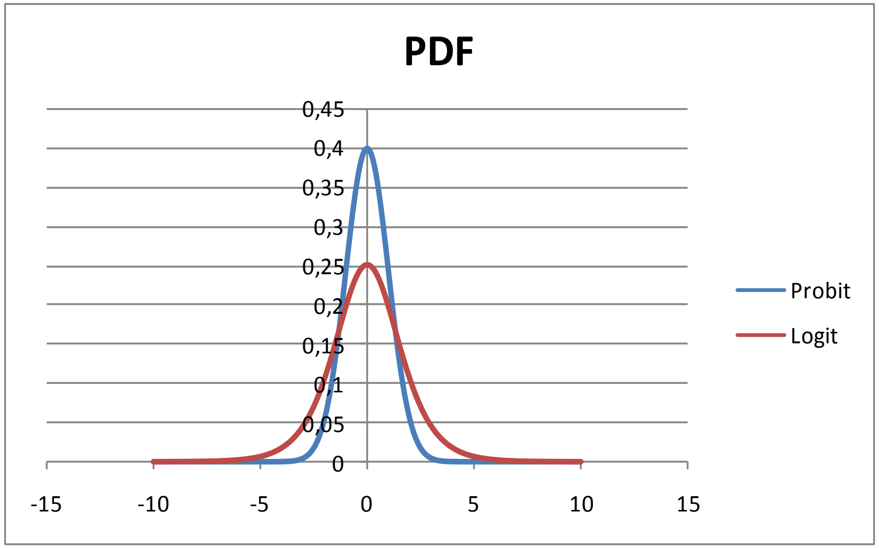
$$p_i = \Pr[y_i = 0 \mid \mathbf{x}_i] = \Pr[u_i + \boldsymbol{\beta}' \cdot \mathbf{x}_i \le 0] = F(-\boldsymbol{\beta}' \cdot \mathbf{x}_i)$$

- If the distribution is assumed to be normal than we get the Probit model
- If the residuals follow the logistic distribution than we get the Logit model

$$F(x) = \Lambda(x) = \frac{e^x}{1+e^x} = \frac{1}{e^{-x}+1}$$

Logit or Probit?





- The models are similar, the tails are heavier for Logit
- Logit model is numerically more efficient
- Its coefficients naturally interpreted – impact on odds or log odds $\dfrac{p_i}{1-p_i} = e^{-\boldsymbol{\beta}'\cdot\mathbf{x}_i}$ $\ln G/B = \ln\dfrac{1-p_i}{p_i} = \boldsymbol{\beta}'\cdot\mathbf{x}_i$

Maximum Likelihood Estimation

- The coefficients could be theoretically estimated by splitting the sample into pools with similar characteristics and by OLS
- …or by maximizing the total likelihood or log-likelihood

$$L(\mathbf{b}) = \prod_i F(-\mathbf{b}'\cdot\mathbf{x}_i)^{y_i}(1-F(-\mathbf{b}'\cdot\mathbf{x}_i))^{1-y_i}$$

$$\ln L(\mathbf{b}) = l(\mathbf{b}) = \sum_i y_i \ln F(-\mathbf{b}'\cdot\mathbf{x}_i) + (1-y_i)\ln(1-F(-\mathbf{b}'\cdot\mathbf{x}_i))$$

Estimators' properties

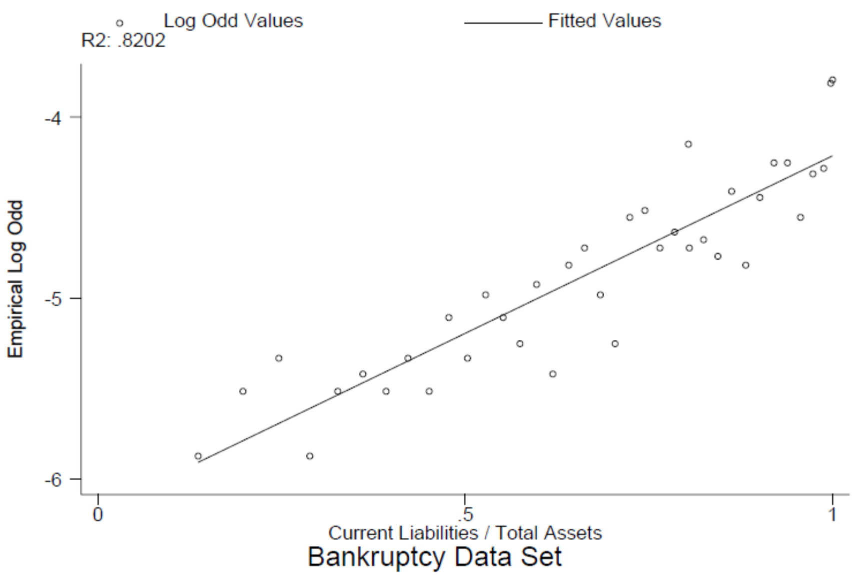
- Hence $\frac{\partial l}{\partial b_j} = \sum_i (y_i - \Lambda(-\mathbf{b}'\cdot\mathbf{x}_i))x_{i,j} = 0 \text{ for } \; j = 0,\ldots,k$
- The solution is found by the Newton's algorithm in a few iterations
- The estimated parameters $\mathbf{b}$ are asymptotic normal and the s.e. are reported by most statistical packages allowing to test the null hypothesis and obtain confidence intervals

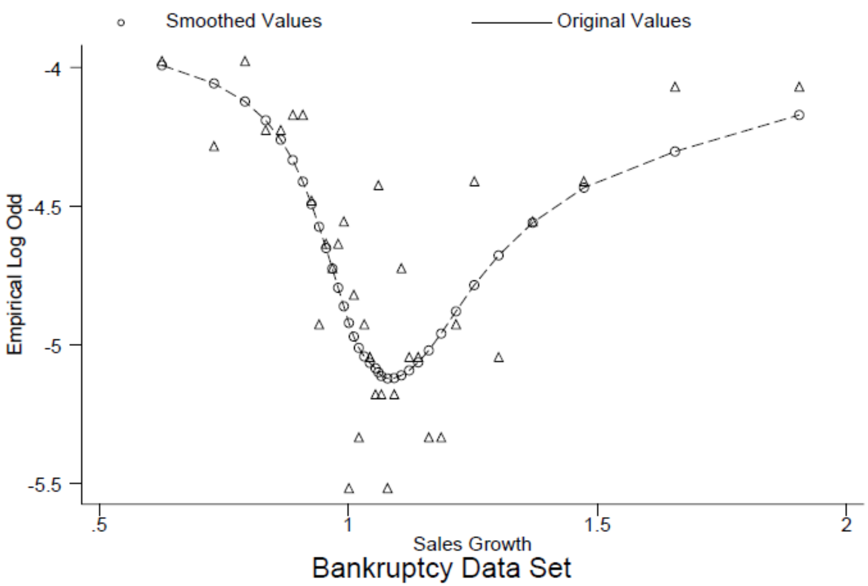
Selection of Variables

- In-sample likelihood is maximized if all possible variables are used
- But insignificant coefficients where we are not sure even with the sign can cause systematic out-sample errors
- The goal is to have all coefficients significant at least on the 90% probability level and at the same time maximize the Gini coefficient

Selection of Variables

- Generally it is useful to perform univariate analysis to preselect the variables and make appropriate transformations





Note that the charts use log B/G odds

Selection of Variables

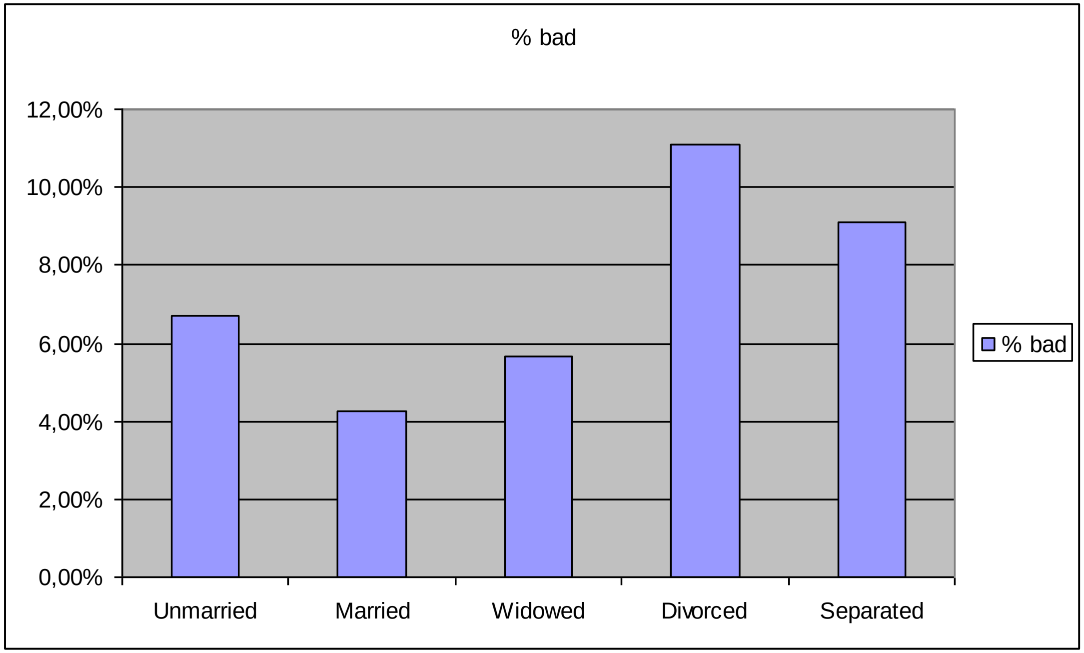
- Correlated variables should be eliminated
- In the backward selection procedure variables with low significance are eliminated
- In the forward selection procedure variables are added maximizing the $\chi^2(k-l)$ statistic

$$G = -2\ln\left(\frac{\text{likelihood of the restricted model}}{\text{likelihood of the larger model}}\right)$$

Categorical Variables

- Some categorical values need to be merged

Marital Status of borrower				
	No. of good loans	No. of bad loans	Total	% bad
Unmarried	1200	86	1286	6,69%
Married	900	40	940	4,26%
Widowed	100	6	106	5,66%
Divorced	800	100	900	11,11%
Separated	60	6	66	9,09%
Total	**1860**	**152**	**2012**	**7,55%**



Categorical Variables

- Discriminatory power can be measured looking on significance of the regression coefficients (of the dummy variables)
- Or using the Weight of Evidence (WoE)

$$WoE = \ln \Pr[c \mid Good] - \ln \Pr[c \mid Bad]$$

- Interpretation follows from the Bayes Theorem

$$\frac{\Pr[Good \mid c]}{\Pr[Bad \mid c]} = \frac{\Pr[c \mid Good]}{\Pr[c \mid Bad]} \cdot \frac{\Pr[Good]}{\Pr[Bad]}$$

Weight of Evidence

Marital Status of borrower			
	Pr[c|Good]	Pr[c|Bad]	WoE
Unmarried	0,64516129	0,565789474	0,13127829
Married or widowed	0,537634409	0,302631579	0,57466264
Divorced or separated	0,462365591	0,697368421	-0,410958
		IV	**0,24204342**

- An overall discriminatory power of the variable is measured by the information value

$$IV = \sum_{c=1}^{C} WoE(c) \cdot \Pr[c \mid Good] - \Pr[c \mid Bad]$$

Weight of Evidence

- WoE can be used to build a naïve Bayes score (based on the independence of categorical variables)

$$\Pr[\mathbf{c} \mid Good] = \Pr[c_1 \mid Good] \cdots \Pr[c_2 \mid Good]$$

$$\Pr[\mathbf{c} \mid Bad] = \Pr[c_1 \mid Bad] \cdots \Pr[c_2 \mid Bad]$$

$$WoE(\mathbf{c}) = WoE(c_1) + \cdots + WoE(c_n)$$

$$s(\mathbf{c}) = s_{Pop} + WoE(c_1) + \cdots + WoE(c_n)$$

- In practice the variables are often correlated
- WoE can be used to replace categorical by continuous variables

Case Study – Scoring Function development

Scoring Dataset	**observations:**	10,936
31 Jul 2009 14:56	**variables:**	21

variable name	**variable description**	**# of categorical values**
id_deal	*Account Number*	*N/A*
def	*Default (90 days, 100 CZK)*	*2*
mesprij	*Monthly Income*	*N/A*
pocvyz	*Number of Dependents*	*7*
ostprij	*Other Income*	*N/A*
k1pohlavi	*Sex*	*2*
k2vek	*Age*	*15*
k3stav	*Marital Status*	*6*
k4vzdel	*Education*	*8*
k5stabil	*Employment Stability*	*10*
k6platce	*Employer Type*	*9*
k7forby	*Type of Housing*	*6*
k8forspl	*Type of Repayment*	*6*
k27kk	*Credit Card*	*2*
k28soczar	*Social Status*	*10*
k29bydtel	*Home Phone Lines*	*3*
k30zamtel	*Employment Phone Lines*	*4*

Case Study – In Sample x Out Sample

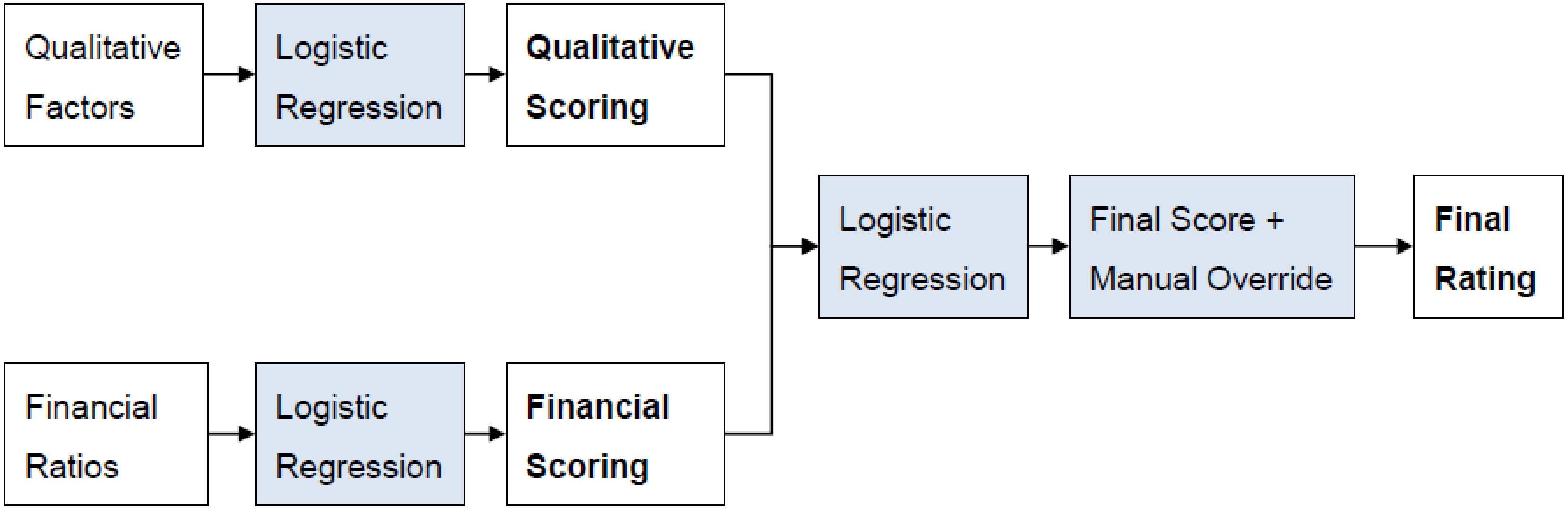
Full Model

Full Model GINI		
Run	**In-sample**	**Out-sample**
1	71.4%	47.9%
2	69.2%	54.7%
3	71.1%	38.8%
4	70.9%	38.4%
5	69.2%	43.6%

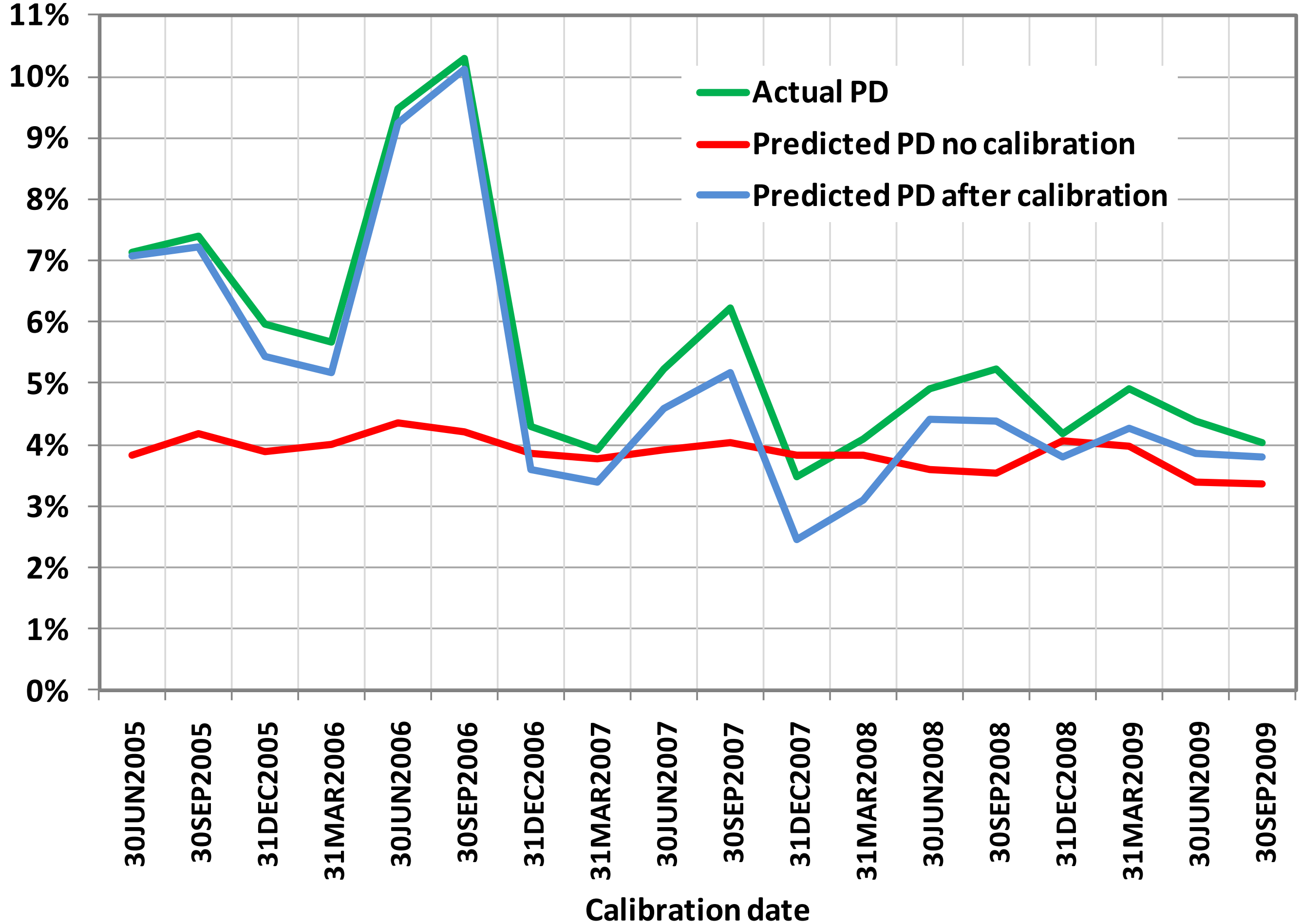
Final Model – coarse classification, selection of variables

Restricted Model GINI		
Run	**In-sample**	**Out-sample**
1	61.9%	57.7%
2	61.3%	58.7%
3	61.6%	58.0%
4	60.0%	62.7%
5	60.7%	59.5%

Combination of Qualitative (Expert) and Quantitative Assesment

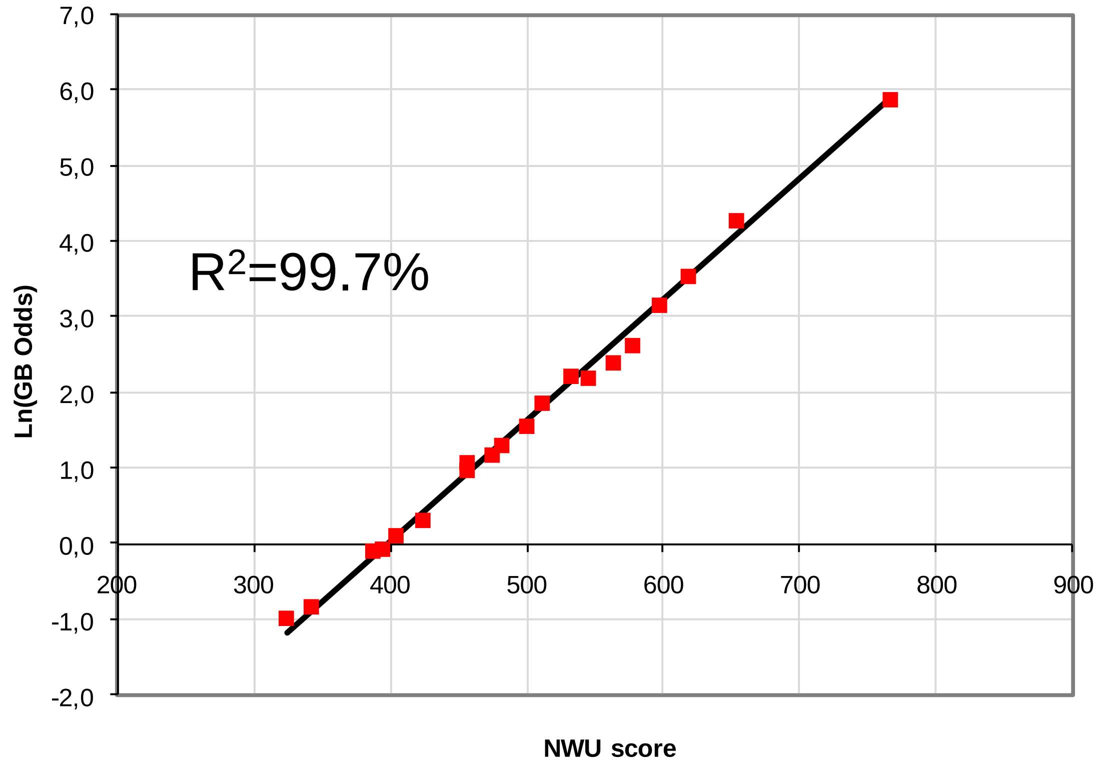


Rating and PD Calibration



Rating and PD Calibration

- Run a regression of ln(G/B) on the score as the only explanatory variable



TTC and PIT Rating/PD

- Point in Time (PIT) rating/PD prediction is based on all available information including macroeconomic situation – desirable from the business point of view
- Through the Cycle (TTC) rating/PD should be on the other hand independent on the cycle – cannot use explanatory variables that are correlated with the cycle – desirable from the regulatory point of view

Shadow Rating

- Not enough defaults for logistic regression but external ratings – e.g. Large corporations, municipalities, etc.
- Regression is run on PDs or log odds obtained from the ratings and with available explanatory variables
- The developed rating mimics the external agency process
- Useful if there is insufficient internal expertise

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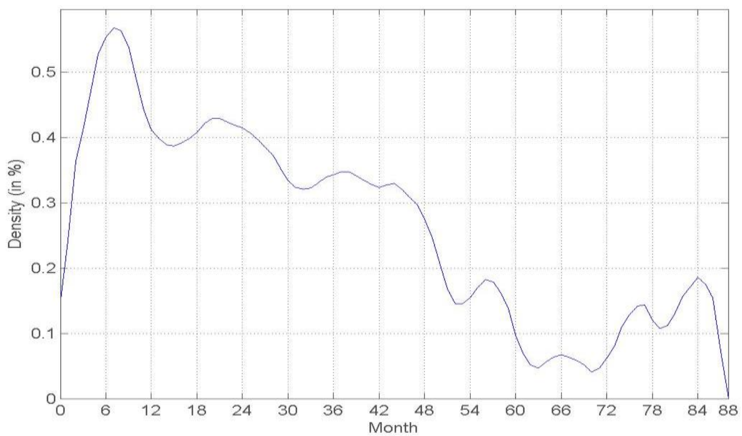
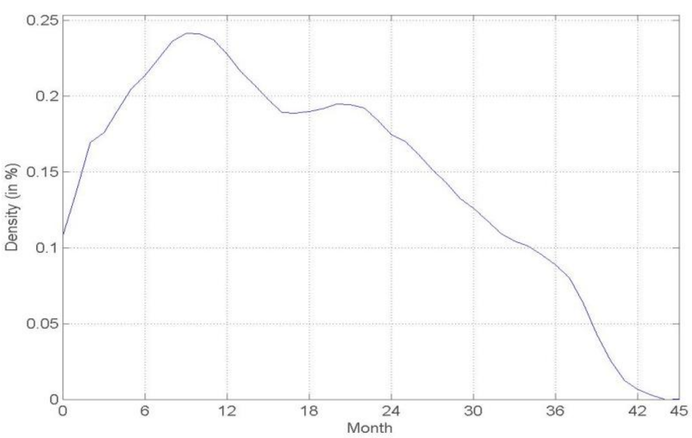
Survival Analysis

So far we have implicitly assumed that the probability of default does not depend on the loan age – empirically it is not the case

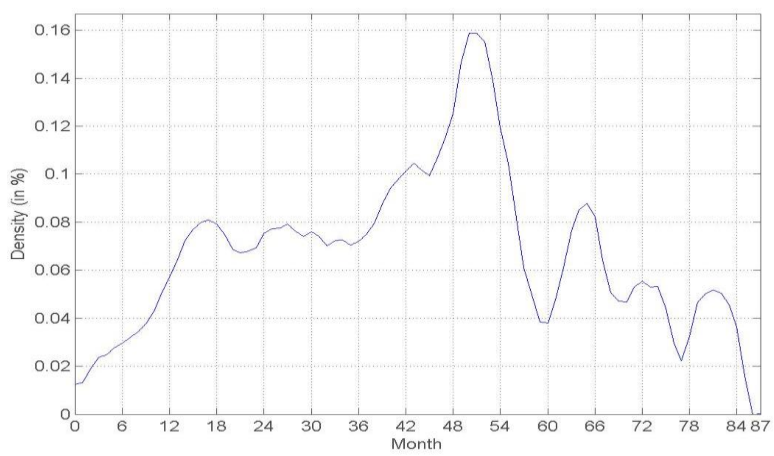
Density of Time to Default from First Drawing - Consumer Loans



Density of Time to Default from First Drawing - Credit Cards



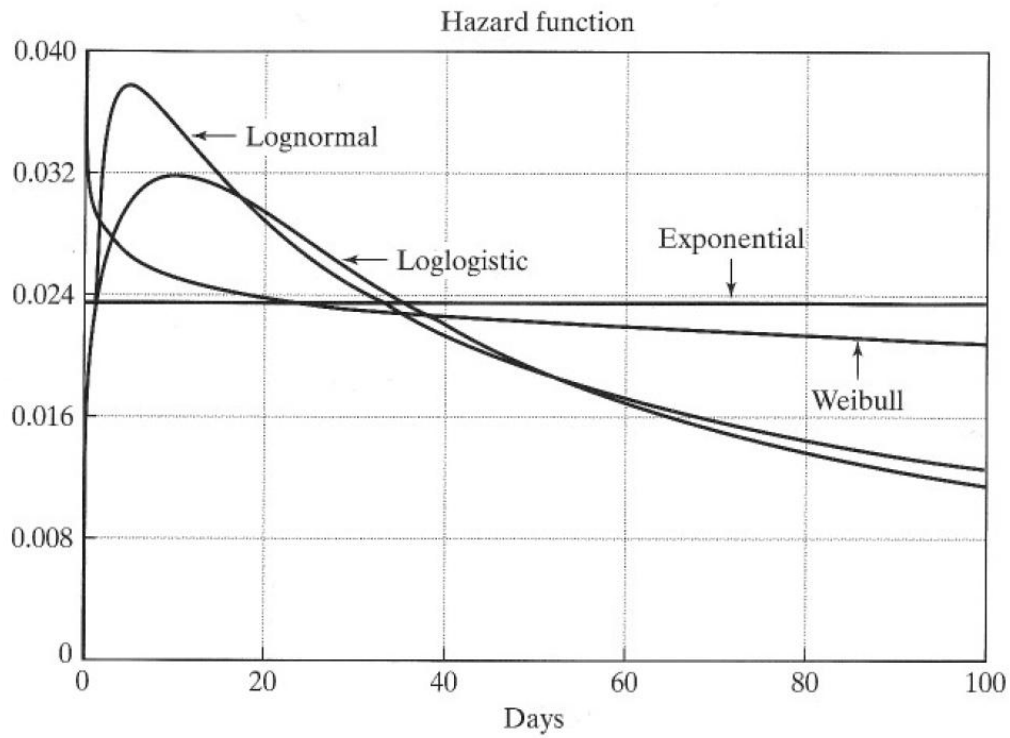
Density of Time to Default from First Drawing - Mortgages and Related C.L.



Survival Analysis

- T - time of exit
- $f(t), F(t)$ - probability density and cumulative distribution functions
- $S(t) = 1 - F(t)$ - survival function
- $\lambda(t) = \dfrac{f(t)}{S(t)}$ - hazard function
- The survival function can be expressed in terms of the hazard function that is modeled primarily

Parametric Hazard Models



Parameters estimated by MLE

$$\ln L(\boldsymbol{\theta}) = \sum_{\substack{\text{uncensored}\\\text{observations}}} \ln \lambda(t \mid \boldsymbol{\theta}) + \sum_{\text{all observations}} \ln S(t \mid \boldsymbol{\theta})$$

Nonparametric Hazard Models

- Cox regression

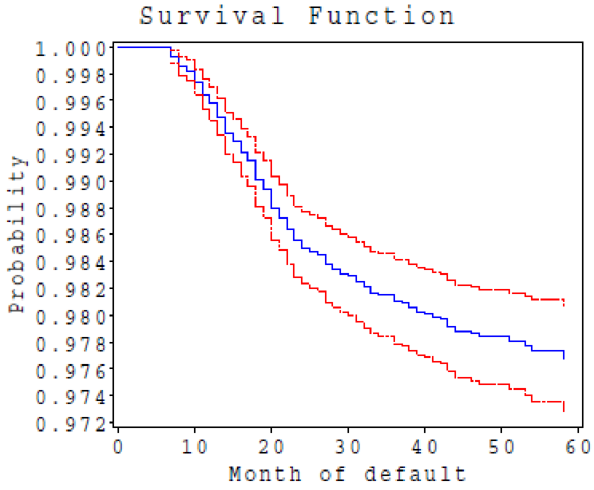
$$\lambda(t,\mathbf{x}) = \lambda_0(t)\exp(-\mathbf{x}'\boldsymbol{\beta}),$$

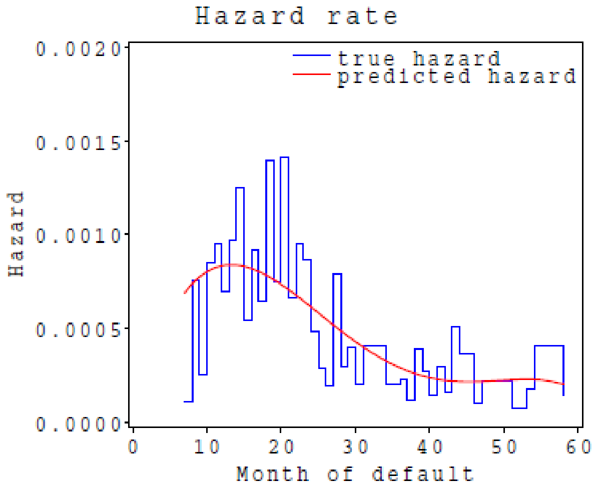
- Baseline hazard function and the risk level function are estimated separately

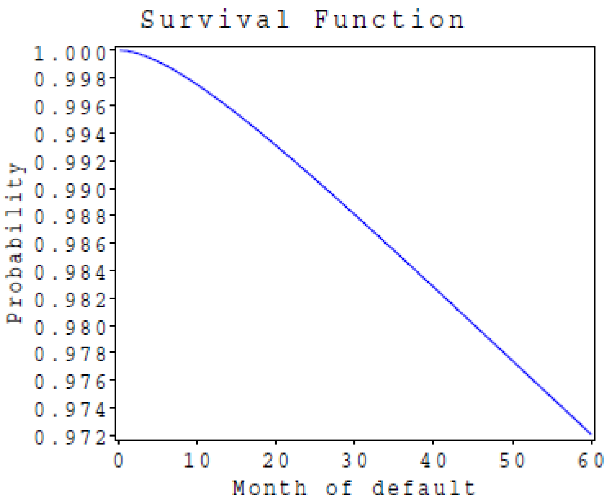
$$L_i(\boldsymbol{\beta}) = \frac{\lambda(t_i,\mathbf{x}_i)}{\sum_{j\in A_i}\lambda(t_j,\mathbf{x}_j)} = \frac{\exp(-\mathbf{x}_i'\boldsymbol{\beta})}{\sum_{j\in A_i}\exp(-\mathbf{x}_j'\boldsymbol{\beta})} \qquad \ln L = \sum_{i=1}^{K}\ln L_i$$

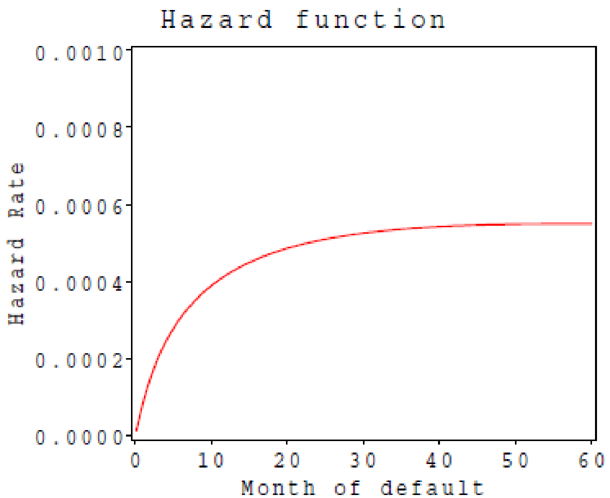
$$L_t = \prod_{i=1}^{n}\left[\lambda_0(t)\exp(\mathbf{x}_i'\boldsymbol{\beta})\right]^{dN_i(t)}\exp\left(-\lambda_0(t)\exp(\mathbf{x}_i'\boldsymbol{\beta})Y_i(t)\right),$$

Cox Regression and an AFT Parametric Model Examples









Markov Chain Models

- Rating transitions driven by a migration matrix with default as a persistent state
- Allows to estimate PDs for longer time horizons

From/To	AAA	AA	A	BBB	BB	B	CCC/C	D	N.R.
AAA	87.44	7.37	0.46	0.09	0.06	0.00	0.00	0.00	4.59
AA	0.60	86.65	7.78	0.58	0.06	0.11	0.02	0.01	4.21
A	0.05	2.05	86.96	5.50	0.43	0.16	0.03	0.04	4.79
BBB	0.02	0.21	3.85	84.13	4.39	0.77	0.19	0.29	6.14
BB	0.04	0.08	0.33	5.27	75.73	7.36	0.94	1.20	9.06
B	0.00	0.07	0.20	0.28	5.21	72.95	4.23	5.71	11.36
CCC/C	0.08	0.00	0.31	0.39	1.31	9.74	46.83	28.83	12.52

Source: Standard & Poor's Global Fixed Income Research; Standard & Poor's CreditPro® 7.0.

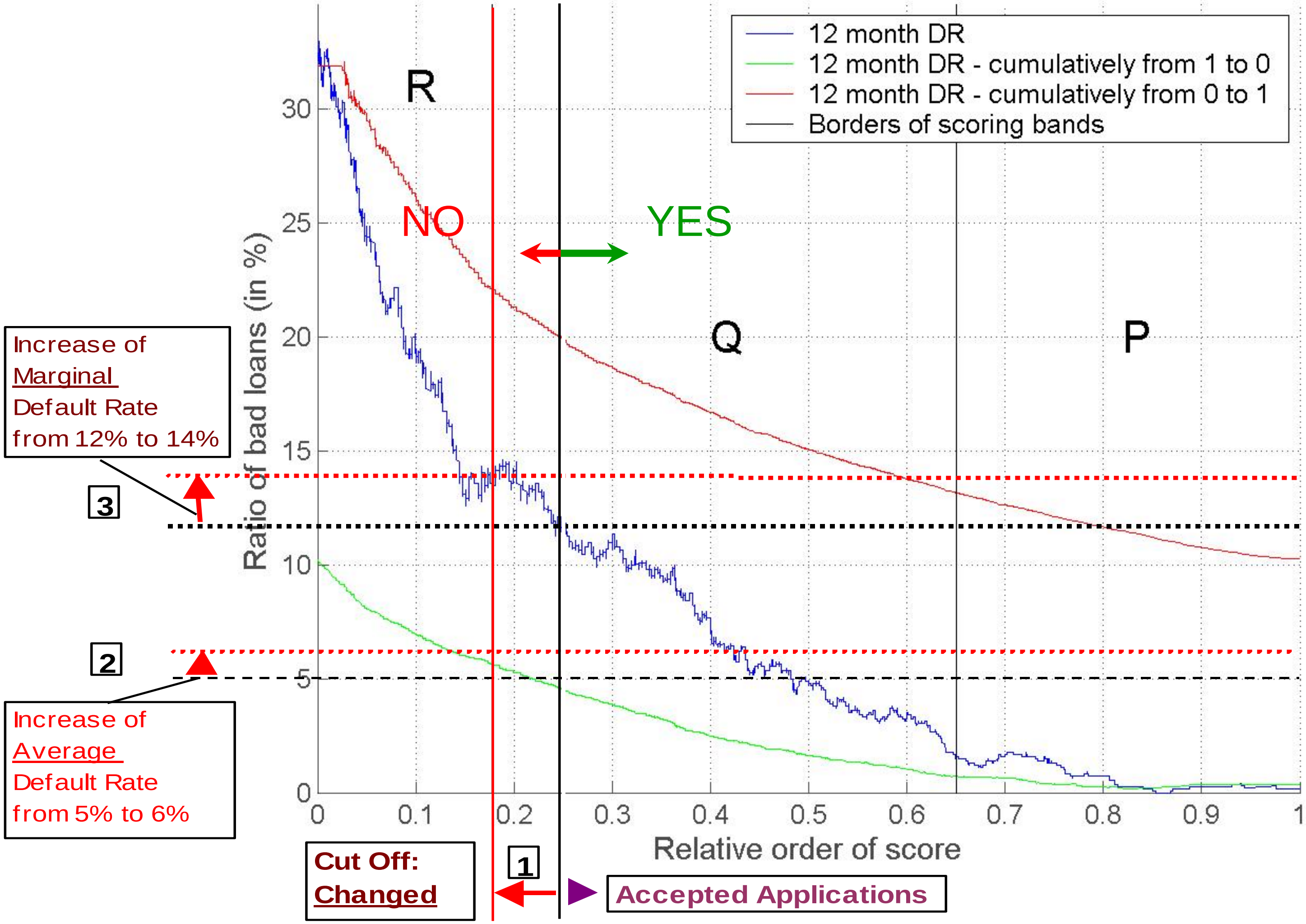
Expected Loss, PD, LGD, and EAD

- Loan interest rate = internal cost of funds + administrative cost + cost of risk + profit margin
- Credit risk cost = annualized expected loss

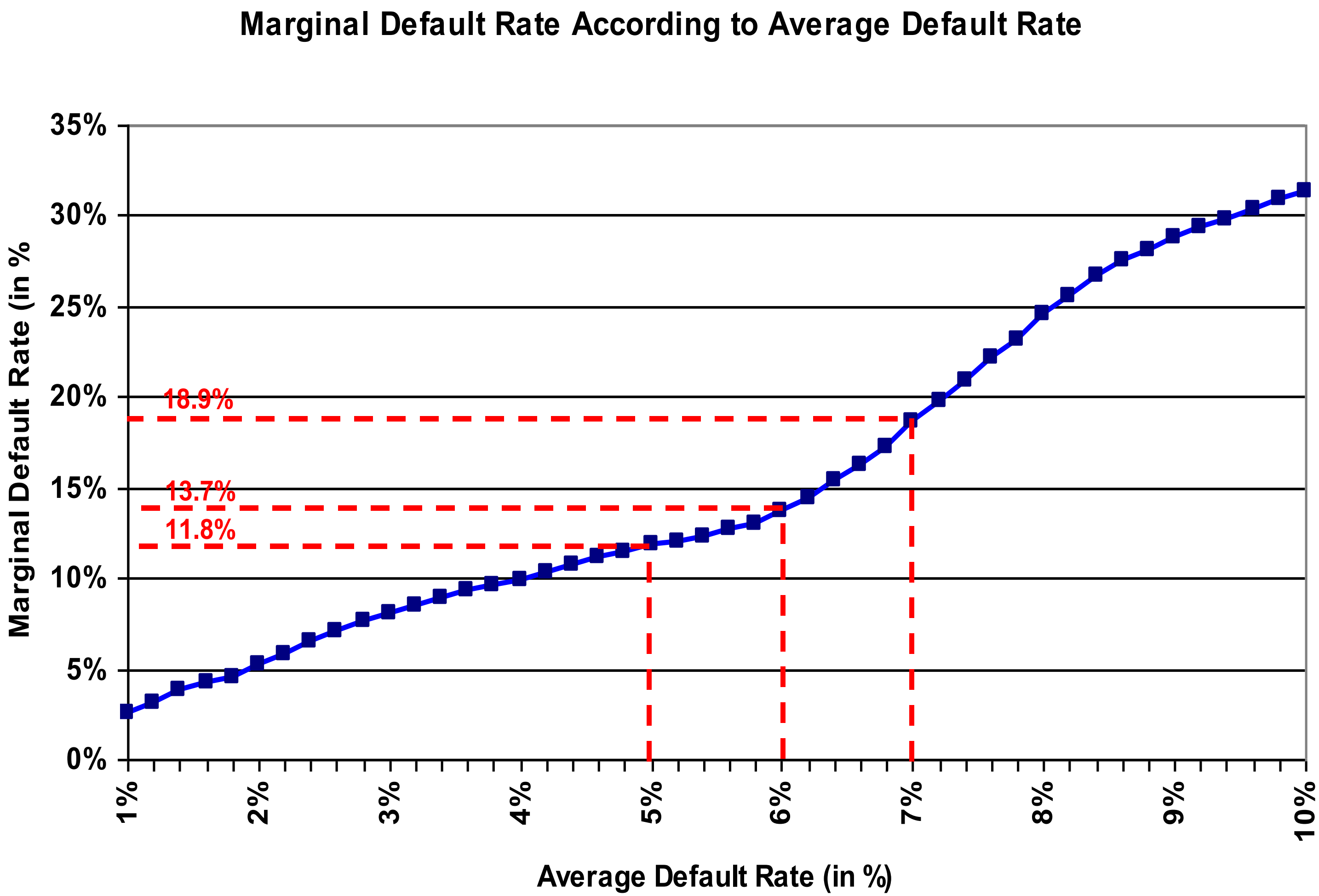
$$RP = \frac{EL}{1 - PD}$$

- $EL^{abs} = PD*LGD*EAD$

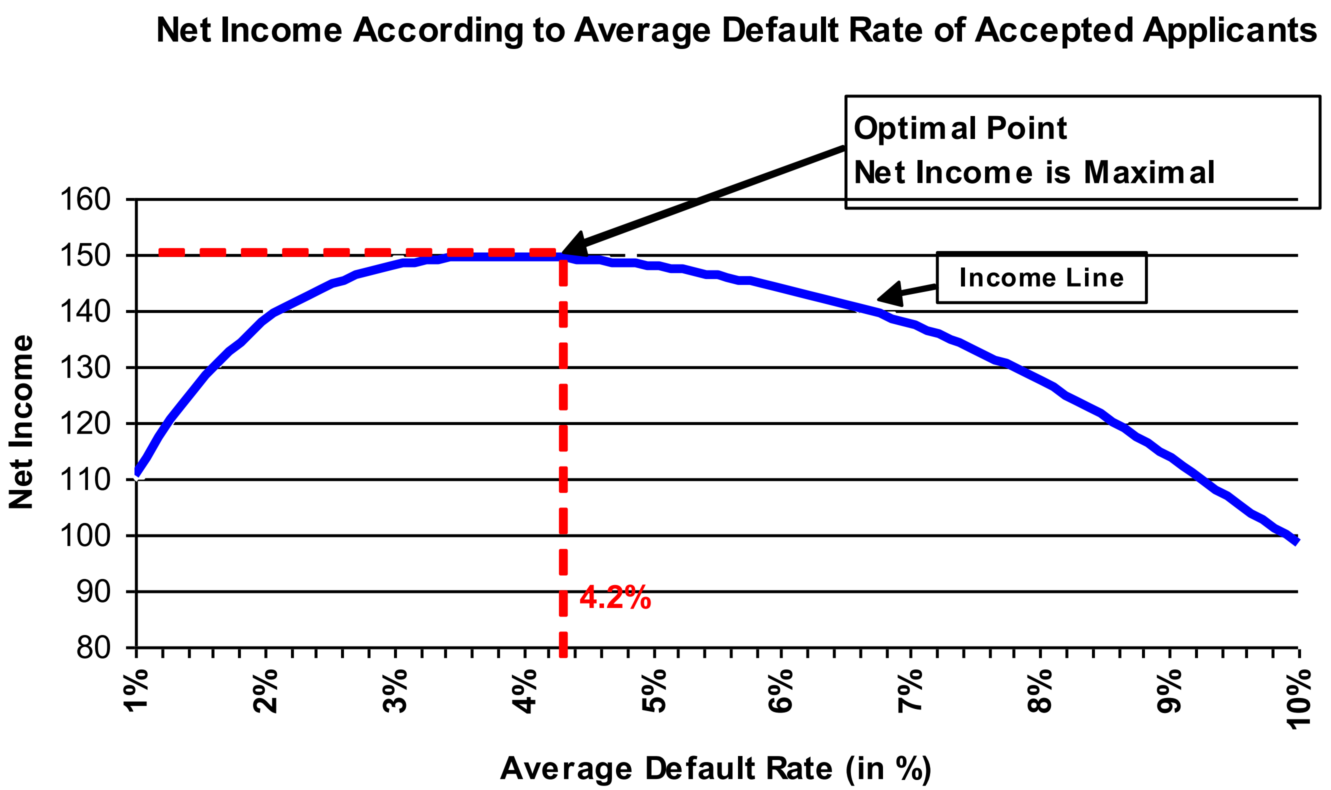
Approval Parametrization



Marginal Risk



Cut-off Optimization



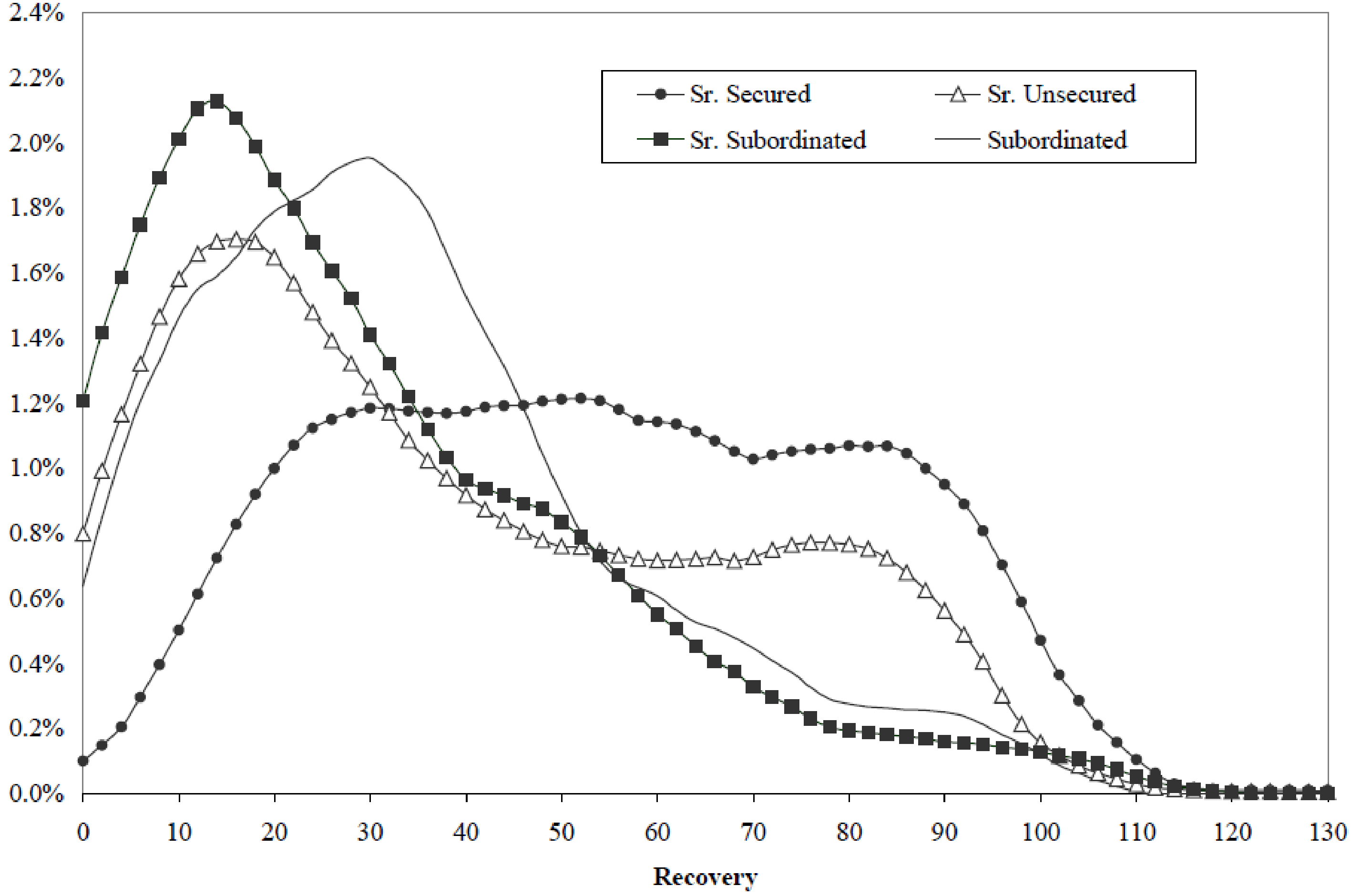
Recovery Rates and LGD Estimation

- LGD = 1 – RR where RR is defined as the market value of the bad loan immediately after default divided by EAD or rather

$$RR = \frac{1}{EAD}\sum_{i=1}^{n}\frac{CF_{t_i}}{(1+r)^{t_i}}.$$

- We must distinguish realized (expost) and expected (ex ante) LGD
- LGD is closely related to provisions and write-offs

Recovery Rate Distribution



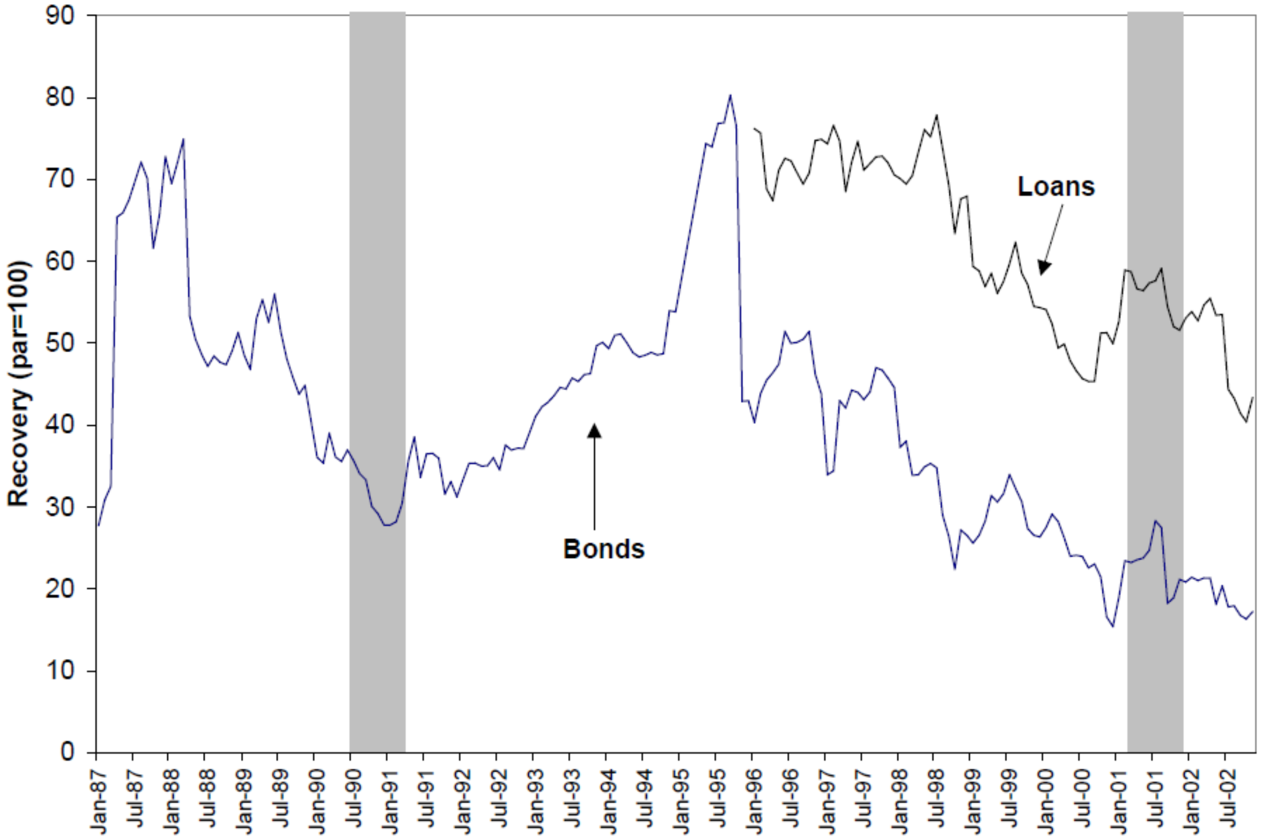
LGD Regression

- Pool level estimations – basic approach
- Advanced: account level regression requires a sufficient number of observations

$$LGD_i = f(\boldsymbol{\beta}'\mathbf{x}_i) + \varepsilon_i$$

- Link function should transform a normal distribution to an appropriate LGD distribution, e.g. Logit, beta or mixed beta distributions

RR Pro-cyclicality



Defaulted bond and bank loan recovery index, U.S. obligors. Shaded regions indicate recession periods. (Source: Schuermann, 2002)

Provisions and Write-offs

- LGD is an expectation of loss on a non defaulted account (regulatory and risk management purpose)
- BEEL is an expectation of loss on a defaulted account
- Provisions reduce on balance sheet value of impaired receivables (IFRS) – created and released – immediate P/L impact
- Write-offs (with/without abandoning) – essentially terminal losses, but positive recovery still possible

Exposure at Default

- What will be the exposure of a loan in case of default within a time horizon?
- Relatively simple for ordinary loans but difficult for lines of credits, credit cards, overdrafts, etc.
- Conversion factor – CAD mandatory parameter

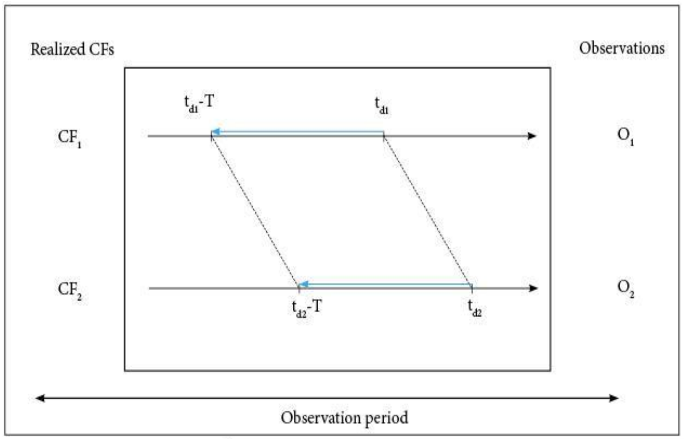
$$EAD = \text{Current Exposure} + CF \cdot \text{Undrawn Limit}$$

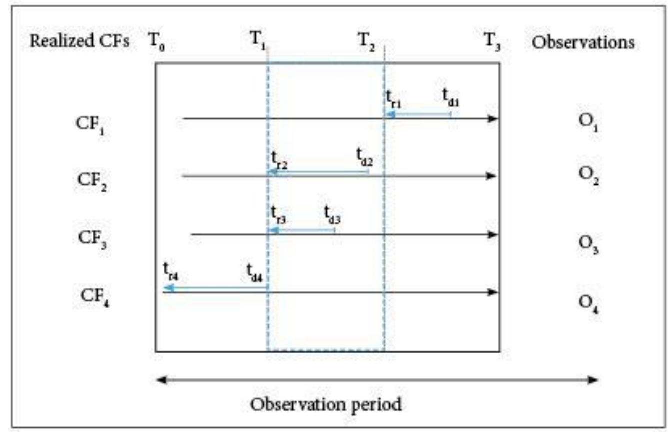
- Ex post calculation requires a reference point observation

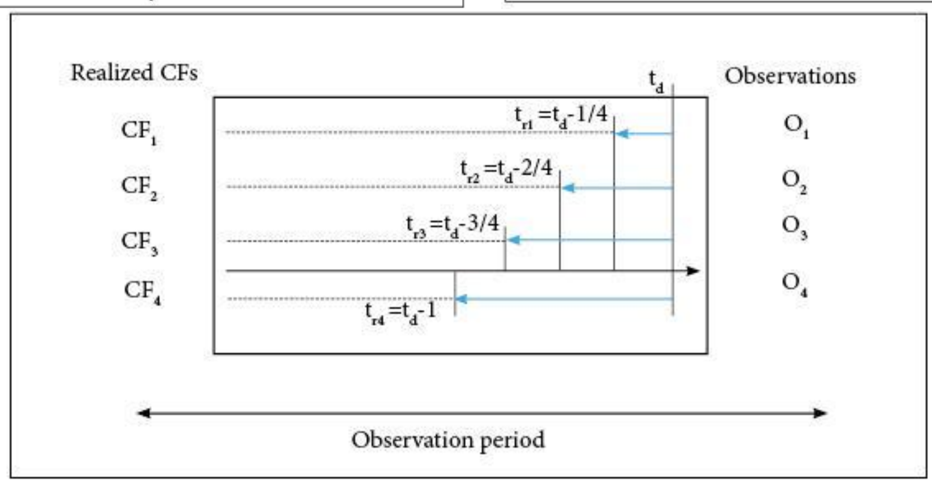
$$CF = CF(a, t_r) = \frac{Ex(t_d) - Ex(t_r)}{L(t_r) - Ex(t_r)}$$

Conversion Factor Dataset

- The CF dataset may be constructed based on a fixed horizon, cohort, or variable time horizon approach







Conversion Factor Estimation

- For very small undrawn amounts CF values do not give too much sense, and so pool based default weighted mean does not behave very well

$$CF(l) = \frac{1}{|RDS(l)|} \sum_{o \in RDS(l)} CF(o)$$

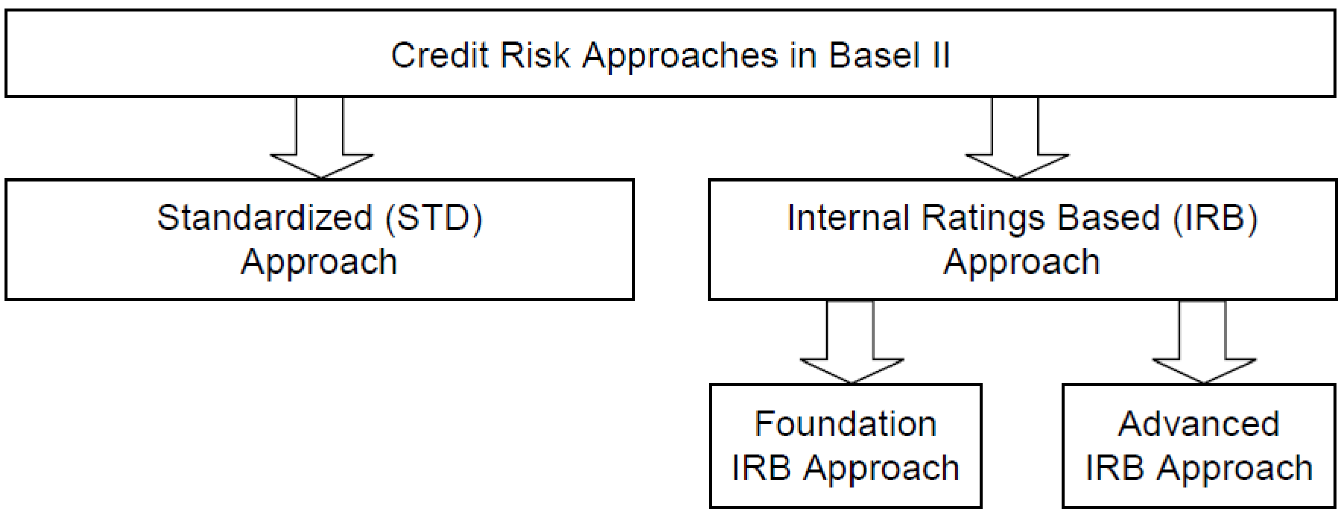
- A weighted or regression based approach recommended

$$CF(l) = \frac{\sum (EAD(o) - Ex(o))}{\sum (L(o) - Ex(o))} \qquad EAD(o) - Ex(o) = \beta(L(o) - E(o)) + \varepsilon$$

$$CF(l) = \frac{\sum (EAD(o) - Ex(o)) \cdot (L(o) - Ex(o))}{\sum (L(o) - Ex(o))^2}$$

Basel II Requirements

$$\text{Capital Ratio} = \frac{\text{Total Capital}}{\text{Credit Risk} + \text{Market Risk} + \text{Operational Risk RWA}}$$



Stadardized Approach (STD)

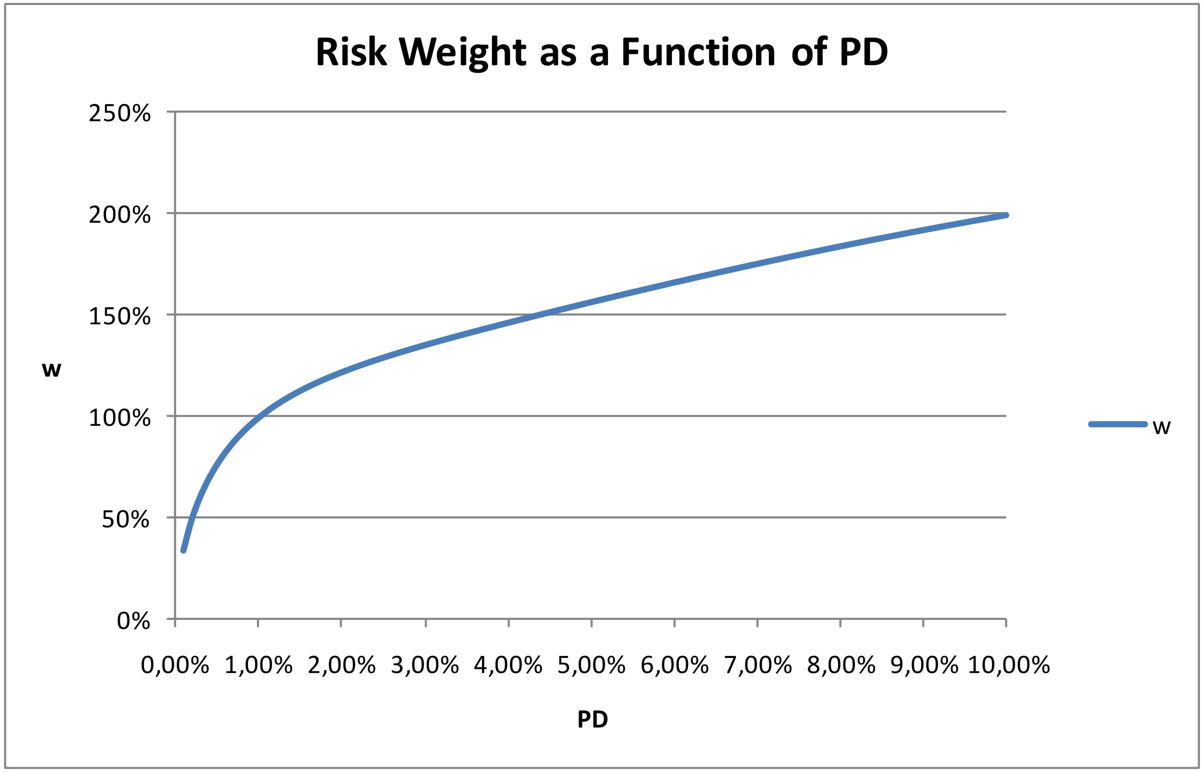
Rating	Sovereign Risk Weights	Bank Risk Weights	Rating	Corporate Risk Weights
AAA to AA-	0%	20%	AAA to AA-	20%
A+ to A-	20%	50%	A+ to A-	50%
BBB+ to BBB-	50%	100%	BBB+ to BBB-	100%
BB+ to B-	100%	100%	BB+ to BB-	100%
Below B-	150%	150%	Below BB-	150%
Unrated	100%	100%	Unrated	100%

Table 1. Risk Weights for Sovereigns, Banks, and Corporates (Source: BCBC, 2006a)

Internal Rating Based Approach (IRB)

$$K = (UDR(PD) - PD) \cdot LGD \cdot MA$$

$$\mathrm{RWA} = EAD \cdot w, \quad w = K \cdot 12.5$$



Basel II IRB Risk weights for corporates (LGD = 45%, M = 3)

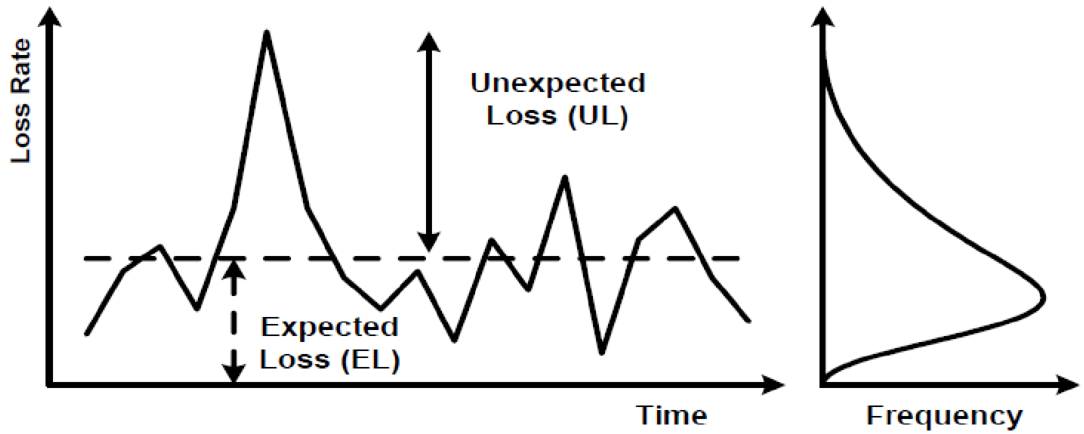
Foundation versus Advanced IRB

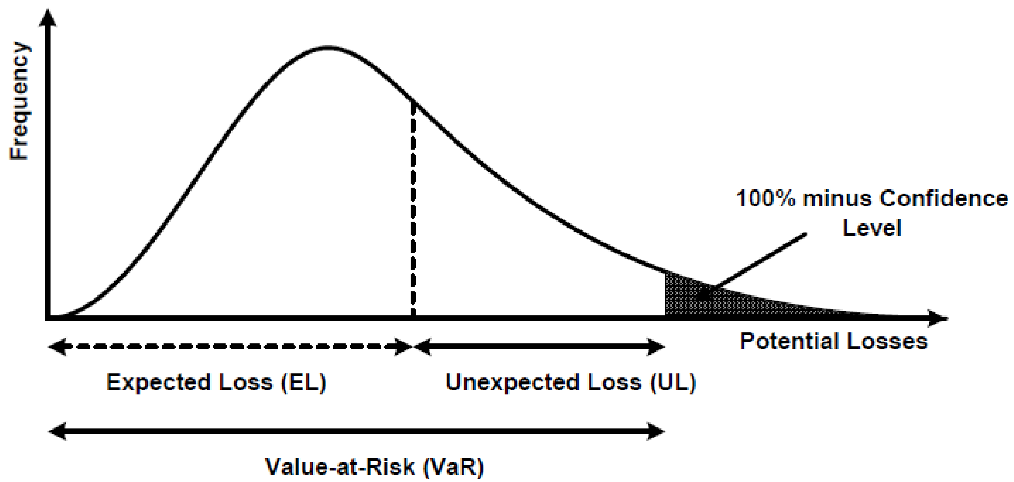
- Foundation approach uses regulatory LGD and CF parameters (table give)
- Advanced approach – own estimates of LGD, CF
- For retail segments only STD or IRBA options possible
- In IRBA BEEL and overall EL must be compared with provisions – negative differences => additional capital requirement

Content

- ✓ Introduction
- ✓ Credit Risk Management
- ✓ Rating and Scoring Systems
- ➢ Portfolio Credit Risk Modeling
- • Credit Derivatives

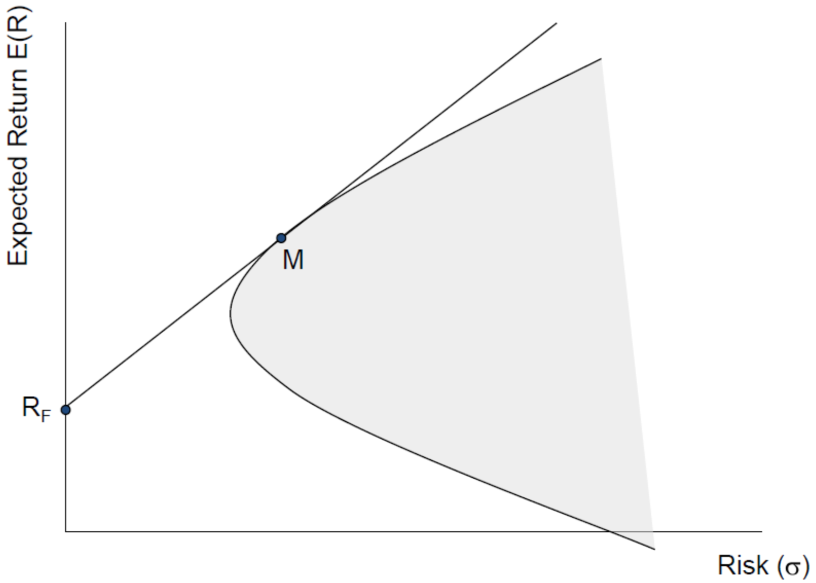
Portfolio Credit Risk Modeling





Markowitz Portfolio Theory

- Can we apply the theory to a loan portfolio?
- Yes, but with economic capital measuring the risk



Expected and Unexpected Loss

- Let *X* denote the loss on the portfolio in a fixed time horizon *T*

$$EL = E[X]$$

$$F_X(x) = \Pr[X \le x]$$

$$q_\alpha^X = \sup\{x \mid F(x) \ge \alpha\}$$

$$UL = q_\alpha^X - E[X]$$

Normality => $UL = VaR_\alpha^{rel} = q_\alpha^N \cdot \sigma$

Credit VaR

- Credit loss of a credit portfolio can be measured in different ways
  - Market value based
  - Accounting provisions
  - Number of defaults x LGD
- Credit VaR at an appropriate probability level should be compared with the available capital => Economic Capital
- Economic Capital can be allocated to individual transactions, implemented e.g. by Bankers Trust
- Many different Credit VaR models! (CreditMetrics, CreditRisk+, CreditPortfolio View, KMV portfolio Manager, Vasicek Model – Basel II)

CreditMetrics

- Well known methodology by JP Morgan
- Monte Carlo simulation of rating migration
- Today/future bond/loan values determined by ratings
- Historical rating migration probabilities
- Rating migration correlations modeled as asset correlations – structural approach
- Asset return decomposed into a systematic (macroeconomic) and idiosyncratic (specific) parts

Bond Valuation

- Simulated forward values required calculation of implied forward discount rates for individual ratings

$$P = \sum_t \frac{CF(t)}{(1+r_s(t))^t}$$

$$1+r_s(t) = (1+r_s(t_0))(1+r_s(t_0,t))$$

$$P_u = \sum_{t \geq t_0} \frac{CF(t)}{(1+r_u(t_0,t))^{t-t_0}}$$

Rating Migrations

One-year transition matrix (%)

Initial Rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	0.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

Source: Standard & Poor's CreditWeek (15 April 96)

Recovery Rates

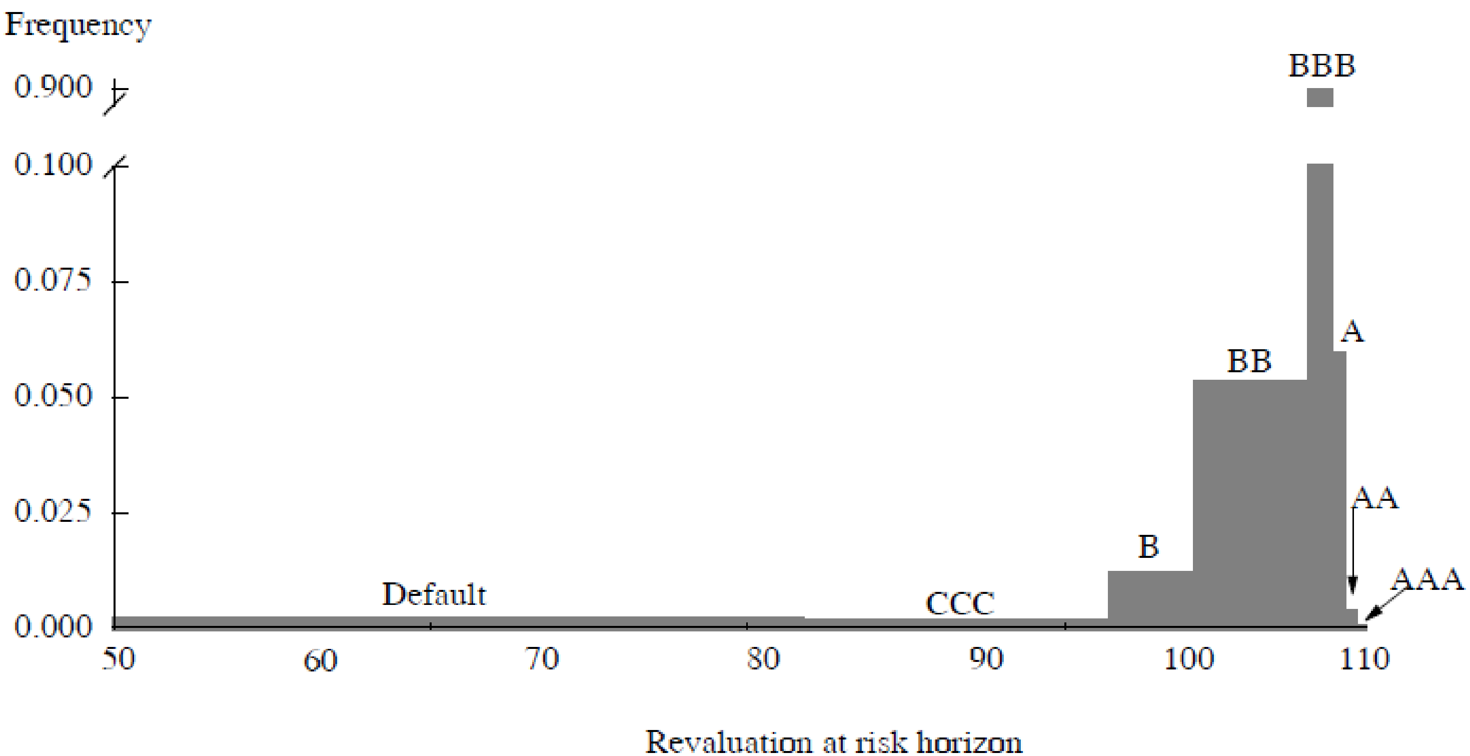
- CreditMetrics models the recovery rates as deterministic or independent on the rate of defaults but a number of studies show a negative correlation

Seniority Class	Mean (%)	Standard Deviation (%)
Senior Secured	53.80	26.86
Senior Unsecured	51.13	25.45
Senior Subordinated	38.52	23.81
Subordinated	32.74	20.18
Junior Subordinated	17.09	10.90

Source: Carty & Lieberman [96a] —Moody's Investors Service

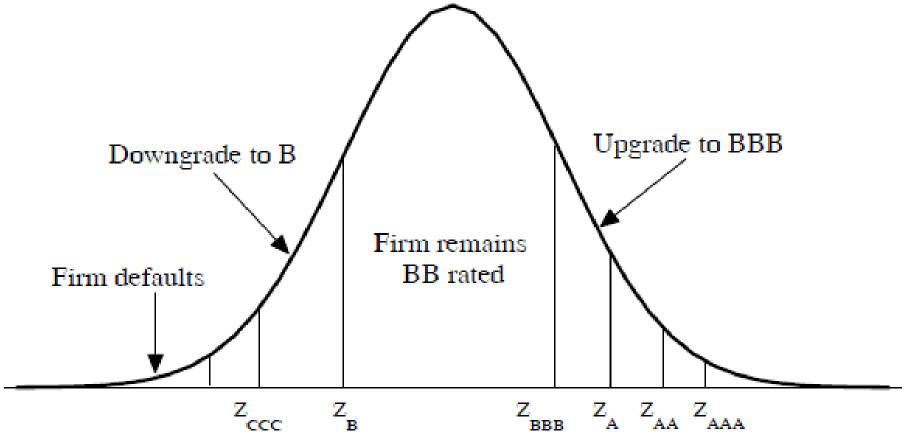
Single Bond Portfolio

Distribution of value for a 5-year BBB bond in one year



Credit Migration (Merton's) Structural Model

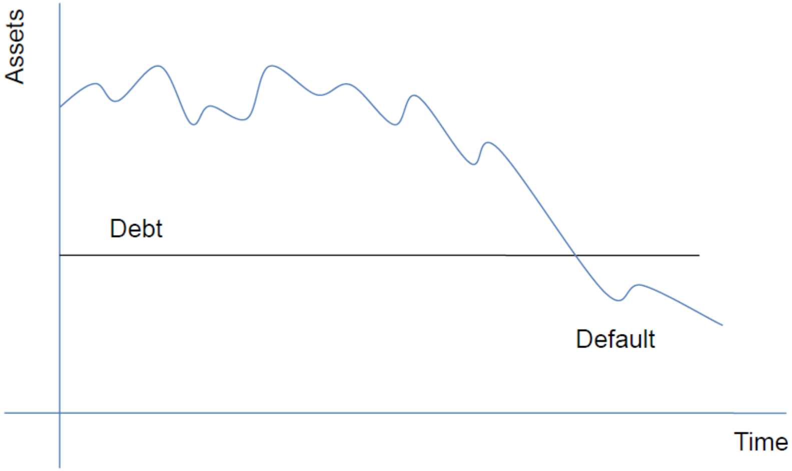
- In order to capture migrations parameterize credit migrations by a standard normal variable which can be interpreted as a normalized change of assets



- The thresholds calibrated to historical probabilities

Credit Migration (Merton's) Structural Model

- Default happens if Assets < Debt
- The market value of debt and equity can be valued using option valuation theory
- Stock returns correlations approximate asset return correlations



Merton's Structural Model

- Creditors' payoff at maturity = risk free debt – European put option

$$D(T) = D - \max(D - A(T), 0)$$

- Shareholders' payoff = European call option

$$E(T) = \max(A(T) - D, 0)$$

- Geometric Brownian Motion

$$dA(t) = \mu A(t)dt + \sigma A(t)dW(t) \qquad d(\ln A) = (\mu - \sigma^2/2)dt + \sigma dW$$

- The rating or its change is determined by the distance to the default threshold or its change, which can be expressed in the standardized form:

$$r = \frac{1}{\sigma}\left(\ln\frac{A(1)}{A(0)} - (\mu - \sigma^2/2)\right) \sim N(0,1)$$

Rating Migration and Asset Correlation

- Asset returns decomposed into a number of systematic and an idiosyncratic factors

$$r_i = \sum_{j=1}^{k} w_j r(I_j) + w_{k+1}\epsilon_i$$

- The decomposition can be based on historical stock price data or on an expert analysis

Asset Correlation - Example

- 90% explained by one systematic factor

$$r_1 = 0.9r(I) + \sqrt{1-0.9^2}\epsilon_1 = 0.9r(I) + 0.44\epsilon_1$$

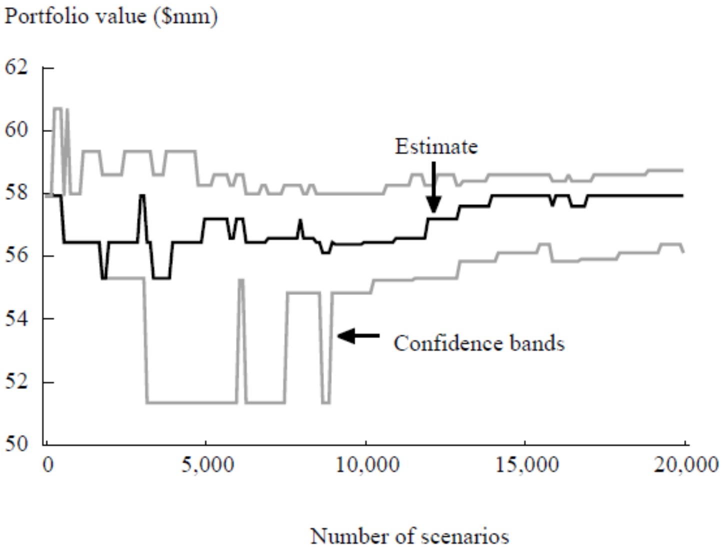
- 70% explained by one systematic factor

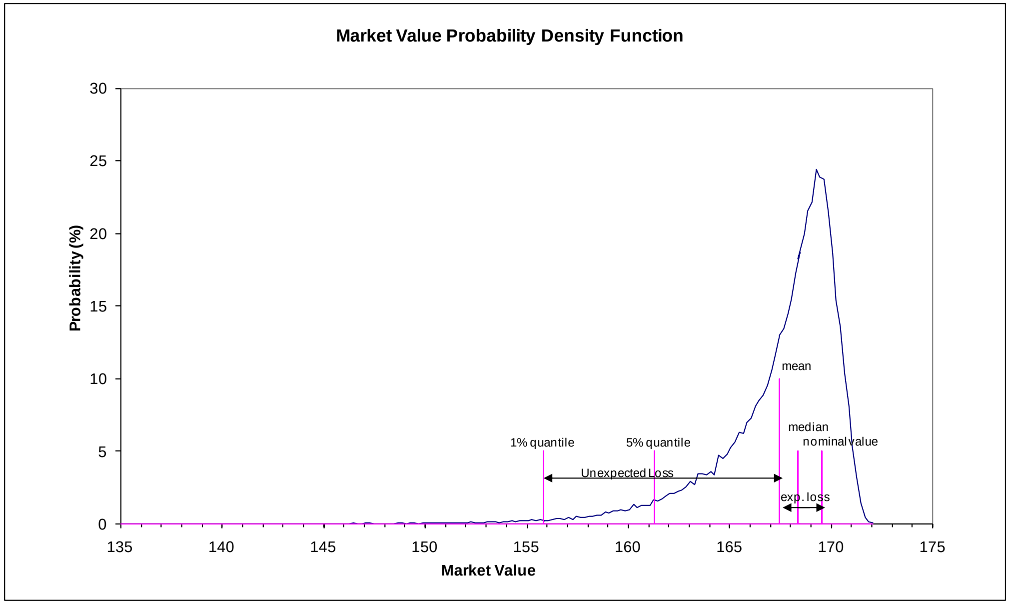
$$r_2 = 0.7r(I) + \sqrt{1-0.7^2}\epsilon_2 = 0.7r(I) + 0.71\epsilon_1$$

- Correlation: $\rho(r_1, r_2) = 0.9 \cdot 0.7 \cdot \rho\left(r(I), r(I)\right) = 0.63$
- More factors – index correlations must be taken into account

Portfolio Simulation

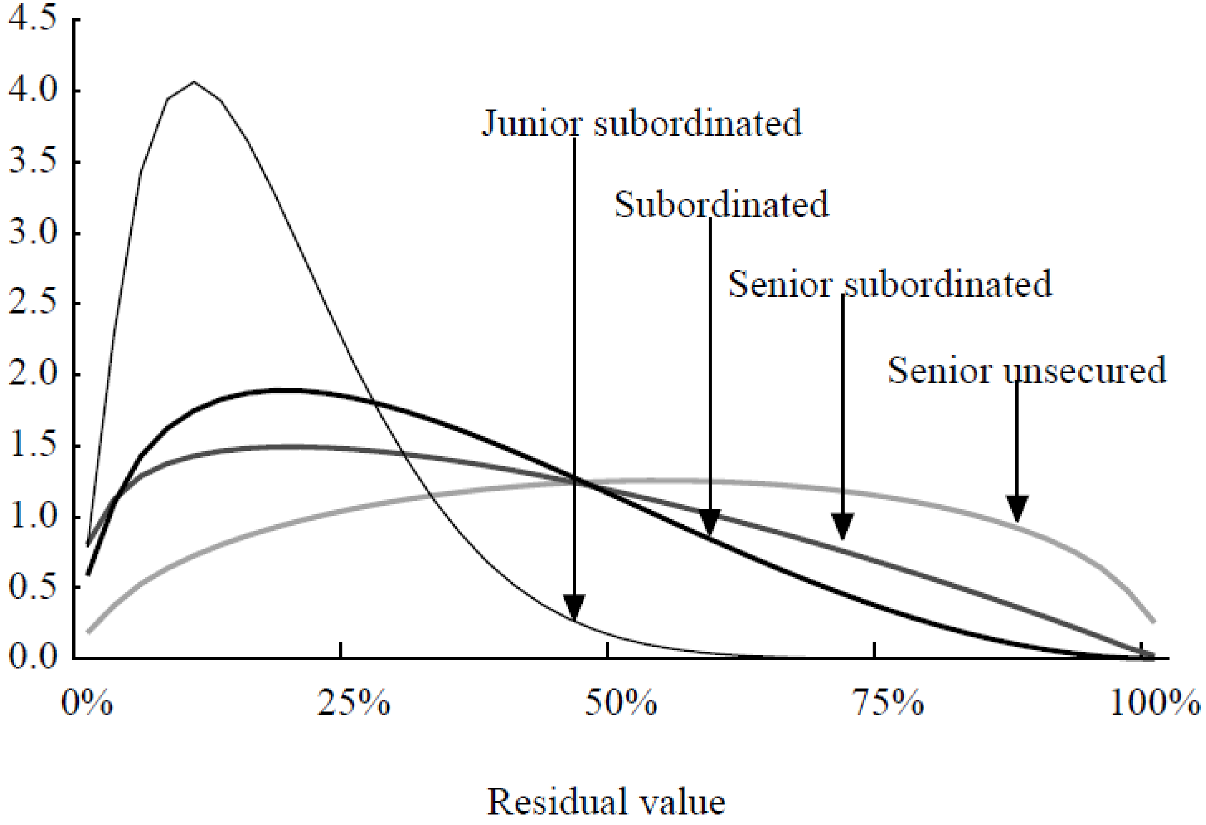
Sample systematic factors (Cholesky decomposition) and the specific factors – determine ratings, calculate implied market values



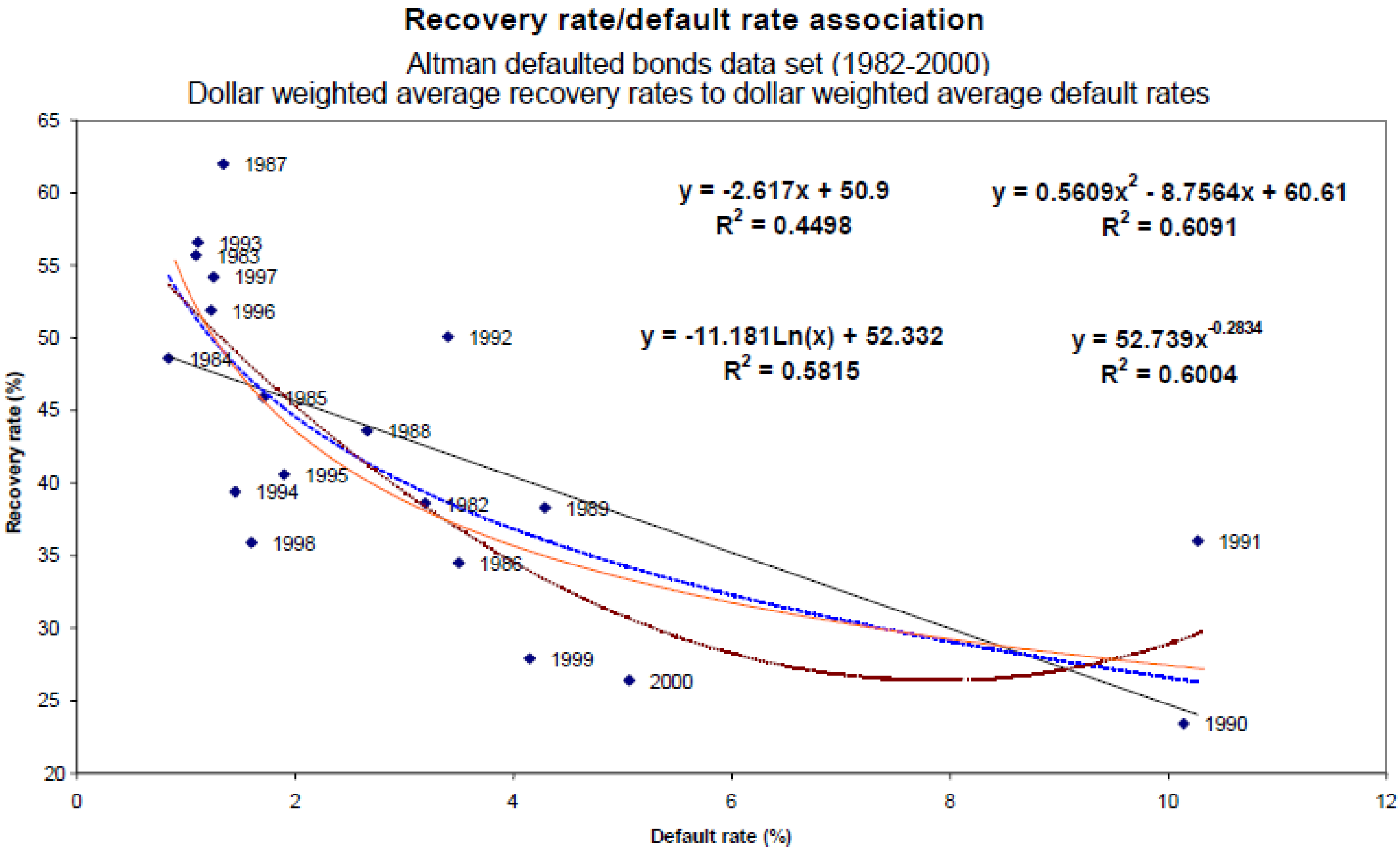


0.1% percentile simulation

Recovery Rates Distribution



Default Rate and Recovery Rate Correlations



CreditRisk+

- Credit Suisse (1997)
- Analytic (generating function) “actuarial” approach
- Can be also implemented/described in a simulation framework:
  1. Starting from an initial PD_0 simulate a new portfolio PD_1
  2. Simulate defaults as independent events with probability PD_1

Credit Risk+
The Full Model

- Exposures are adjusted by fixed LGDs
- The portfolio needs to be divided into size bands (discrete treatment)
- The full model allows a scale of ratings and PDs – simulating overall mean number of defaults with a Gamma distribution
- The portfolio can be divided into independent sector portfolios

Credit Risk+ Details

- Probability generating function

$$X \in \{0,1,2,3,\ldots\} \qquad p_n = \Pr[X = n] \qquad G_X(z) = \sum_{n=0}^{\infty} p_n z^n$$

- Sum of two variables … product of the generating functions

$$G_X(z) \cdot G_Y(z) = \left(\sum_{n=0}^{\infty} p_n z^n\right) \cdot \left(\sum_{n=0}^{\infty} q_n z^n\right) = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} p_i q_{n-i}\right) z^n = G_{X+Y}(z)$$

- Poisson distribution approximating the number of defaults with mean $\mu = N \cdot p$

$$G(z) = e^{\mu(z-1)} = \sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} z^n$$

Credit Risk+ Details

- For more rating grades just put $\mu = \sum_{r=1}^{R} \mu_r$ as

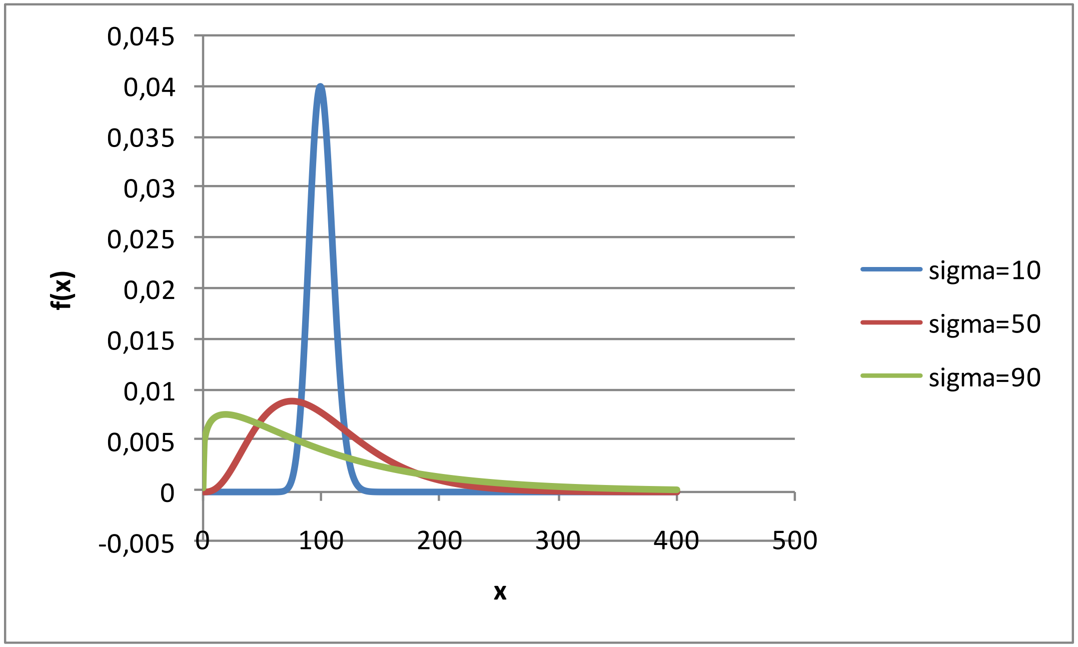
$$e^{\mu_1(z-1)} e^{\mu_2(z-1)} = e^{(\mu_1+\mu_2)(z-1)}$$

- The Poisson distribution can be analytically combined with a Gamma distribution generating the number of defaults (i.e. overall PD)

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} e^{-\frac{x}{\beta}} x^{\alpha-1}, \text{ where } \Gamma(\alpha) = \int_0^{\infty} e^{-x} x^{\alpha-1} dx.$$

Credit Risk+ Details

- Gamma distribution parameters α and β can be calibrated to $\sigma_0 = N \cdot \sigma_{PD}$ and $\mu_0 = N \cdot PD_0$



Credit Risk+ Details

- The distributions of defaults can be expressed as

$$\Pr[X=n]=\int_0^\infty \Pr[X=n\,|\,\mu=x]\,f(x)\,dx$$

- And the corresponding generating function

$$G(z)=\int_0^\infty e^{x(z-1)}f(x)\,dx=\frac{1}{\beta^{\alpha}(1+\beta^{-1}-z)^{\alpha}}$$

$$\Pr[X=n]=(1-q)^{\alpha}\binom{n+\alpha-1}{n}q^{n},\text{where } q=\frac{\beta}{1+\beta}.$$

Credit Risk+ Details

- To obtain the loss distribution we assume that the exposures are multiples of *L* and

$$G(z)=\sum_{n=0}^{\infty}\Pr[\text{portfolio loss}=n\cdot L]\cdot z^{n}$$

- Individual (RR adjusted) exposures have size $m_j \cdot L$ where $j=1,\ldots,k$ hence

$$G_j(z)=e^{\mu_j(z^{m_j}-1)}=\sum_{n=0}^{\infty}e^{-\mu_j}\frac{\mu_j^n}{n!}z^{m_j n}$$

$$G(z)=\prod_{j=1}^{k}G_j(z)=\prod_{j=1}^{k}e^{\mu_j(z^{m_j}-1)}=e^{\sum\mu_j+\sum\mu_j z^{m_j}}=e^{\mu(P(z)-1)}$$

where

$$\mu=\sum_{j=1}^{k}\mu_j$$

$$P(z)=\sum_{j=1}^{k}\frac{\mu_j}{\mu}z^{m_j}$$

Credit Risk+ Details

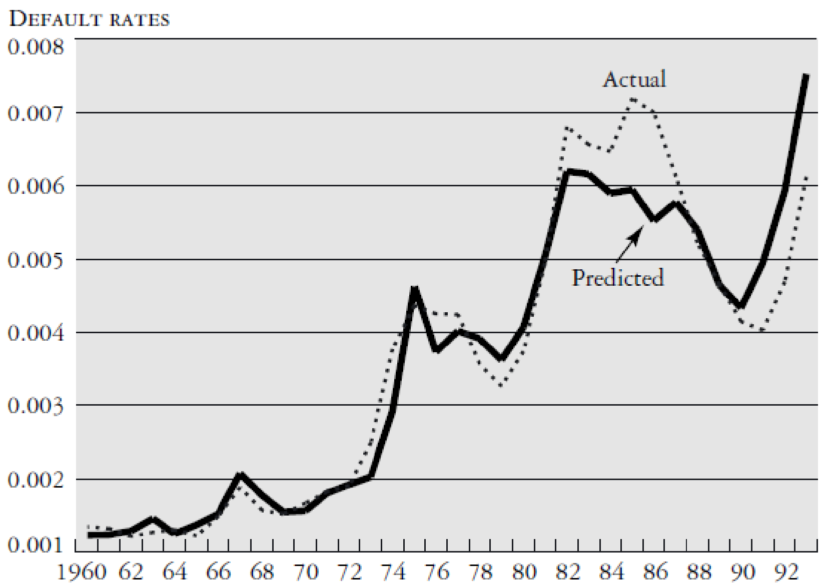
- Finally the generating function needs to be combined with the Gamma distribution

$$G(z) = \int_0^\infty e^{x(P(z)-1)} f(x)dx = \frac{1}{\beta^\alpha (1+\beta^{-1} - P(z))^\alpha}$$

- Here μ_j implicitly adjusts to $\mu \frac{\mu_{0,j}}{\mu_0}$
- For more sector we need to assume independence (!?) to keep the solution analytical $G_{\text{final}}(z) = \prod_{s=1}^{S} G_s(z)$

CreditPortfolio View

- Wilson (1997) and McKinsey
- Ties rates of default to macroeconomic factors



CreditPortfolio View

- Macroeconomic model

$$PD_{j,t} = \Lambda(Y_{j,t})$$

$$Y_{j,t} = \boldsymbol{\beta}_j{}'\mathbf{X}_{j,t} + \epsilon_{j,t}$$

$$X_{j,t,i} = \gamma_{j,i,0} + \gamma_{j,i,0} X_{j,t-1,i} + \gamma_{j,i,0} X_{j,t-2,i} + e_{j,t,i}$$

- Simulate $PD_{j,t}, PD_{j,t+1}, \ldots$ based on information given at $t-1$
- Adjust rating transition matrix to $M_{j,t} = M \quad PD_{j,t} / PD_0$ and simulate rating migrations

KMV Portfolio Manager

- Based on KMV EDF methodology
- Relies on stock market data
- Idea: there is a one/to/one relationship between stock price and its volatility E_0, σ_E and (latent) asset value and its volatility A_0, σ_A

$$A_0 = h_1\left(E_0, \sigma_E\right) \quad \text{and} \quad \sigma_A = h_2\left(E_0, \sigma_E\right)$$

- The asset value and volatility determines the risky claim value $P(0) = f(A_0, \sigma_A)$

An Overview of the KMV Model

1. Based on equity data determine initial asset values and volatilities
2. Estimate asset return correlations
3. Simulate future asset values for a time horizon H and determine the portfolio value distribution

$$P(H) = f(A(H), \sigma_A)$$

- Under certain assumptions there is an analytical solution

Estimation of the Asset Value and Asset Volatility

$$E(T) = \max(A(T) - D, 0)$$

$$dA = rAdt + \sigma_A A dW$$

$$E_0 = A_0 N(d_1) - De^{-rT} N(d_2)$$

$$d_1 = \frac{\ln A_0 / D + (r + \sigma_A^2 / 2)T}{\sigma_A \sqrt{T}} \text{ and } d_2 = d_1 - \sigma_A \sqrt{T}$$

$$dE = \left(\frac{\partial E}{\partial A} rA + \frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} \sigma_A^2 A^2 \right) dt + \frac{\partial E}{\partial A} \sigma_A A dW \qquad \frac{\partial E}{\partial A} = N(d_1)$$

$$\sigma_E = \frac{A_0}{E_0} N(d_1) \sigma_A$$

$$\boxed{E_0 = f_1(A_0, \sigma_A), \sigma_E = f_2(A_0, \sigma_A)}$$

Distance to Default and EDF

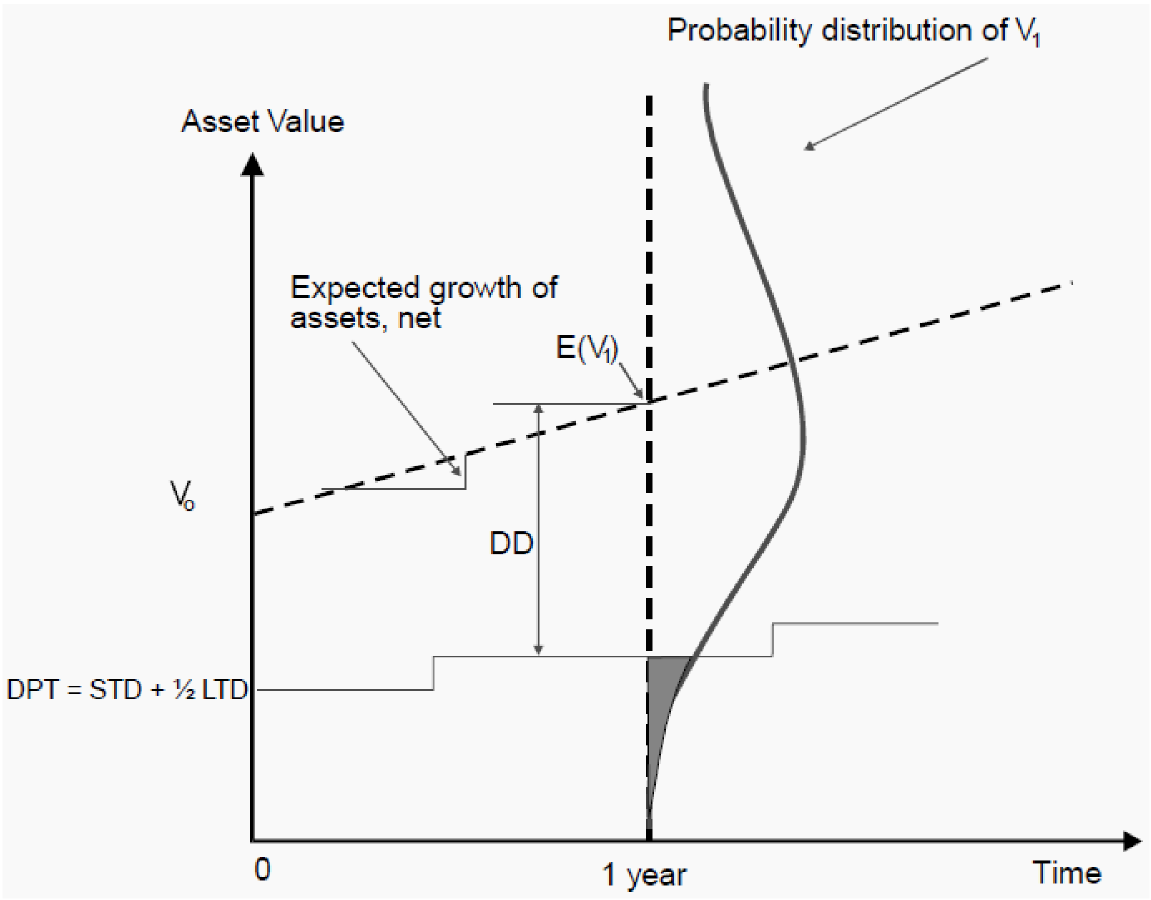
- If just D is payable at T then the risk neutral probability of default would be

$$Q_T = \Pr[V_T < K] = \Phi(-d_2)$$

- But since the capital structure is generally complex we use it or in fact $DD = d_2$, the distance to default in one year horizon as a “score” setting

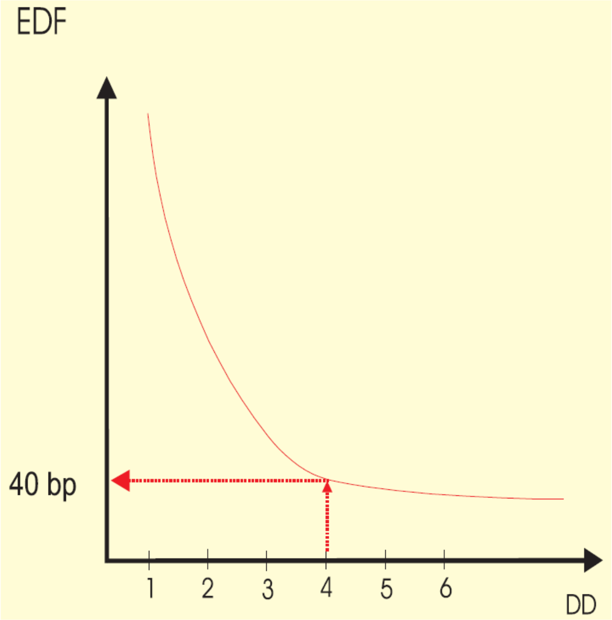
$$DPT = STD + LTD / 2$$

Distance to Default and EDF

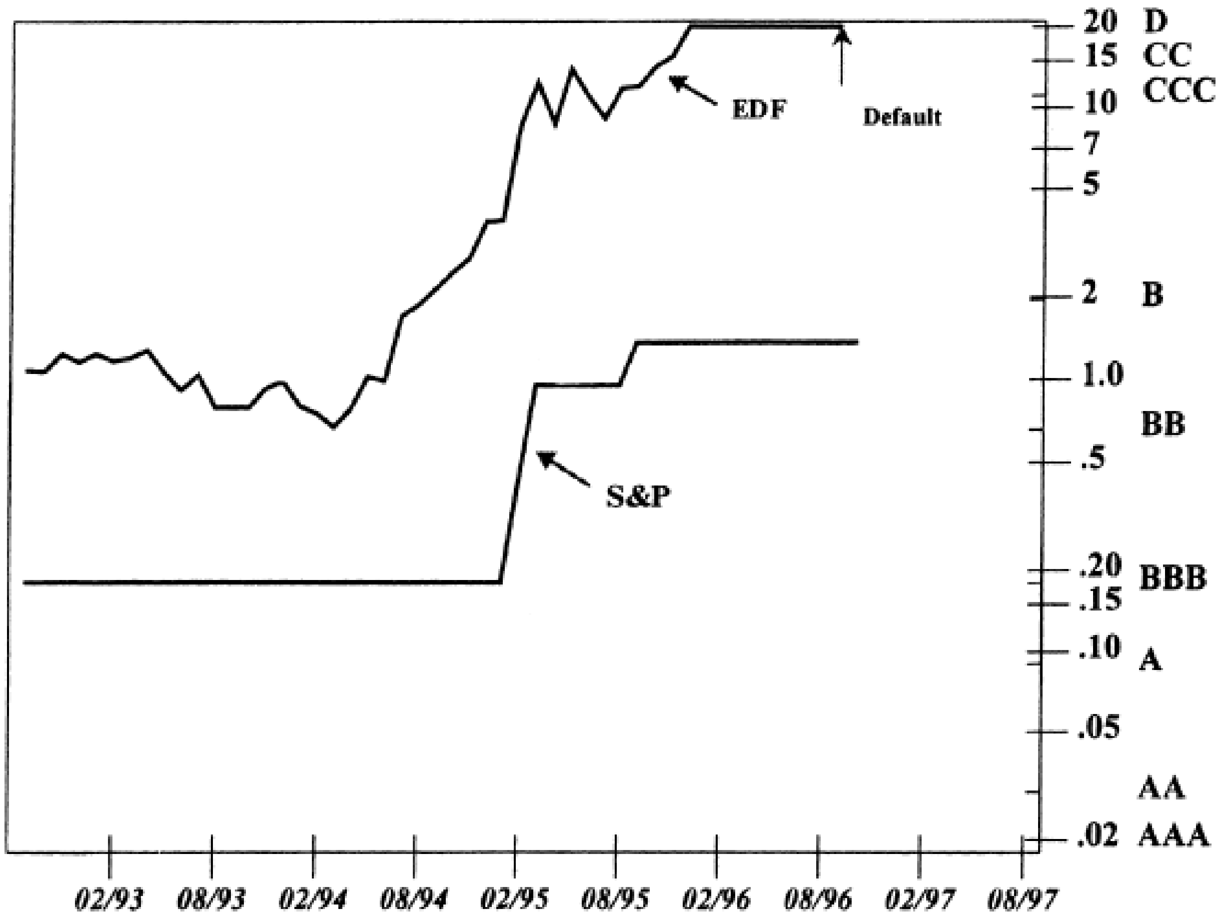


PD callibration

- The distance to default is calibrated to PDs based on historical default observations



EDF of a firm versus S&P rating



KMV versus S&P rating transition matrix

KMV 1-year transition matrix based on non-overlapping EDF ranges[a]

Initial rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	66.26	22.22	7.37	2.45	0.86	0.67	0.14	0.02
AA	21.66	43.04	25.83	6.56	1.99	0.68	0.20	0.04
A	2.76	20.34	44.19	22.94	7.42	1.97	0.28	0.10
BBB	0.30	2.80	22.63	42.54	23.52	6.95	1.00	0.26
BB	0.08	0.24	3.69	22.93	44.41	24.53	3.41	0.71
B	0.01	0.05	0.39	3.48	20.47	53.00	20.58	2.01
CCC	0.00	0.01	0.09	0.26	1.79	17.77	69.94	10.13

[a] *Source*: KMV Corporation.

Transition matrix based on actual rating changes[a]

Initial rating	Rating at year-end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	90.81	8.33	0.68	0.06	0.12	0	0	0
AA	0.70	90.65	7.79	0.64	0.06	0.14	0.02	0
A	0.09	2.27	91.05	5.52	0.74	0.26	0.01	0.06
BBB	0.02	0.33	5.95	86.93	5.30	1.17	1.12	0.18
BB	0.03	0.14	0.67	7.73	80.53	8.84	1.00	1.06
B	0	0.11	0.24	0.43	6.48	83.46	4.07	5.20
CCC	0.22	0	0.22	1.30	2.38	11.24	64.86	19.79

[a] *Source*: Standard & Poor's CreditWeek (April 15, 1996).

Risk neutral probabilities

- The calibrated PDs are historical, but we need risk neutral in order to value the loans

$$PV = E_Q[\text{discounted cash flow}] = \sum_{i=1}^{n} e^{-r_i T_i} E_Q[\text{cash flow}_i] =$$

$$= \sum_{i=1}^{n} e^{-r_i T_i} CF_i (1 - LGD) + \sum_{i=1}^{n} e^{-r_i T_i} (1 - Q_i) CF_i LGD$$

Example

Time	CFi	Qi	e^-rTi	PV1	PV2	PV
1	5 000,00	3%	0,9704	1 940,89	2 824,00	4 764,89
2	5 000,00	6,50%	0,9418	1 883,53	2 641,65	4 525,18
3	105 000,00	9,90%	0,9139	38 385,11	51 877,48	90 262,59
Total				42 209,53	57 343,12	99 552,65

Adjustment of the real world EDFs

$$dA^* = \mu A^* dt + \sigma A^* dW \qquad EDF_T = \Pr[A^*(T) < K_T]$$

$$dA = rAdt + \sigma AdW \qquad Q_T = \Pr[A(T) < K_T]$$

$$EDF_T = N(-d_2),\; d_2 = \frac{\ln A_0 / K_T + (\mu - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$Q_T = N(-d_2^*),\; d_2^* = \frac{\ln A_0 / K_T + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_2^* - (\mu - r)\sqrt{T}/\sigma \qquad Q_T = N\left(N^{-1}(EDF_T) + \frac{\mu - r}{\sigma}\sqrt{T}\right)$$

CAPM: $\mu - r = \beta\pi \qquad \pi = \mu_M - r$

Vasicek's Model and Basel II

- Relatively simple, analytic, asymptotic portfolio with constant asset return correlation
- Let T_j be the time to default of an obligor j with the probability distribution Q_j then $X_j = N^{-1}(Q_j(T_j))$ corresponds to the standardized asset return and $T_j \leq 1$ iff

  $X_j \leq N^{-1}(PD)$
- Hence $PD = Q_j(1)$

Vasicek's Formula

- Let us decompose the X to a systematic and an idiosyncratic part and express the PD conditional on systematic variable $X_j = \sqrt{\rho} \cdot M + \sqrt{1-\rho} \cdot Z_j$

$$PD_1(m) = \Pr\left[T_j \le 1 \mid M = m\right] = \Pr\left[X_j \le N^{-1}(PD) \mid M = m\right] =$$

$$= \Pr\left[\sqrt{\rho} \cdot M \sqrt{1-\rho} \cdot Z_j \le N^{-1}(PD) \mid M = m\right] =$$

$$= \Pr\left[Z_j \le \frac{N^{-1}(PD) - \sqrt{\rho} m}{\sqrt{1-\rho}}\right] = N\left(\frac{N^{-1}(PD) - \sqrt{\rho} m}{\sqrt{1-\rho}}\right)$$

Vasicek's Formula

- So the unexpected default rate on a given probability level $1-\alpha$ is

$$UDR_{\alpha}(PD) = N\left(\frac{N^{-1}(PD)+\sqrt{\rho}\cdot N^{-1}(\alpha)}{\sqrt{1-\rho}}\right)$$

- If multiplied by a constant LGD and EAD we get a "simple" estimate of unexpected loss attributable to a single receivable

Basel II Capital Formula

$$K = (UDR_{99.9\%}(PD) - PD)\cdot LGD\cdot MA$$

$$RWA = EAD\cdot w,\ w = K\cdot 12.5$$

- Correlation is given by the regulation and depends on the exposure class, e.g. for corporates

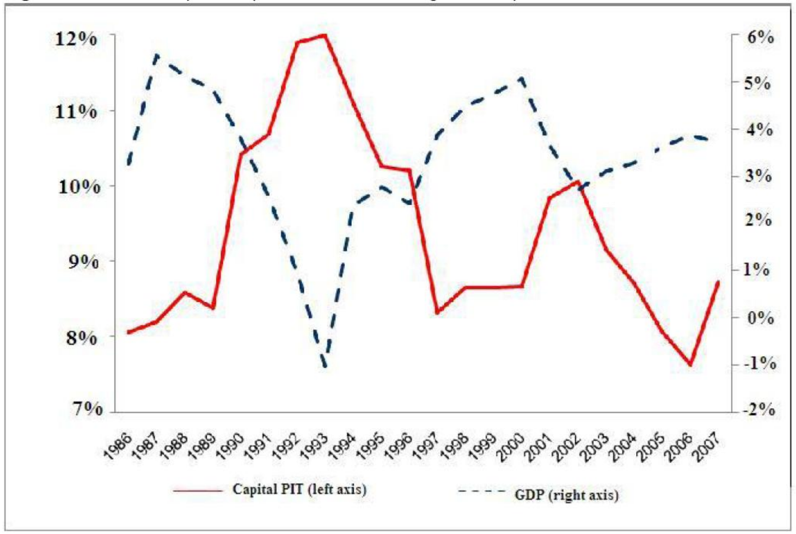
$$\rho = 0.12\frac{1-e^{-50PD}}{1-e^{-50}} + 0.24\frac{e^{-50PD} - e^{-50}}{1-e^{-50}}$$

- Similarly the maturity adjustement

$$MA = \frac{1+(M-2.5)b}{1-1.5b},\quad b = (0.1182 - 0.05478\cdot \ln PD)^2$$

Basel II Problems

- Vague modeling of unexpected LGD
- The issue of PDxLGDxEAD correlation
- Sensitivity to the definition of default
- Procyclicality!!!



Basel III ?!

- Recent BCBS proposals reacting to the crisis
- Principles for Sound Liquidity Risk Management and Supervision (9/2008)
- International framework for liquidity risk measurement, standards and monitoring - consultative document (12/2009)
- Consultative proposals to strengthen the resilience of the banking sector announced by the Basel Committee (12/2009)

Basel III ?!

- Comments could be sent by 16/4/2010 (viz www.bis.org)
- Final decision approved 12/2010
- Key elements
  - Stronger Tier 1 capital
  - Higher capital requirements on counterparty risk
  - Penalization for high leverage and complex models dependence
  - Counter-cyclical capital requirement
  - Provisioning based on expected losses
  - Minimum liquidity requirements

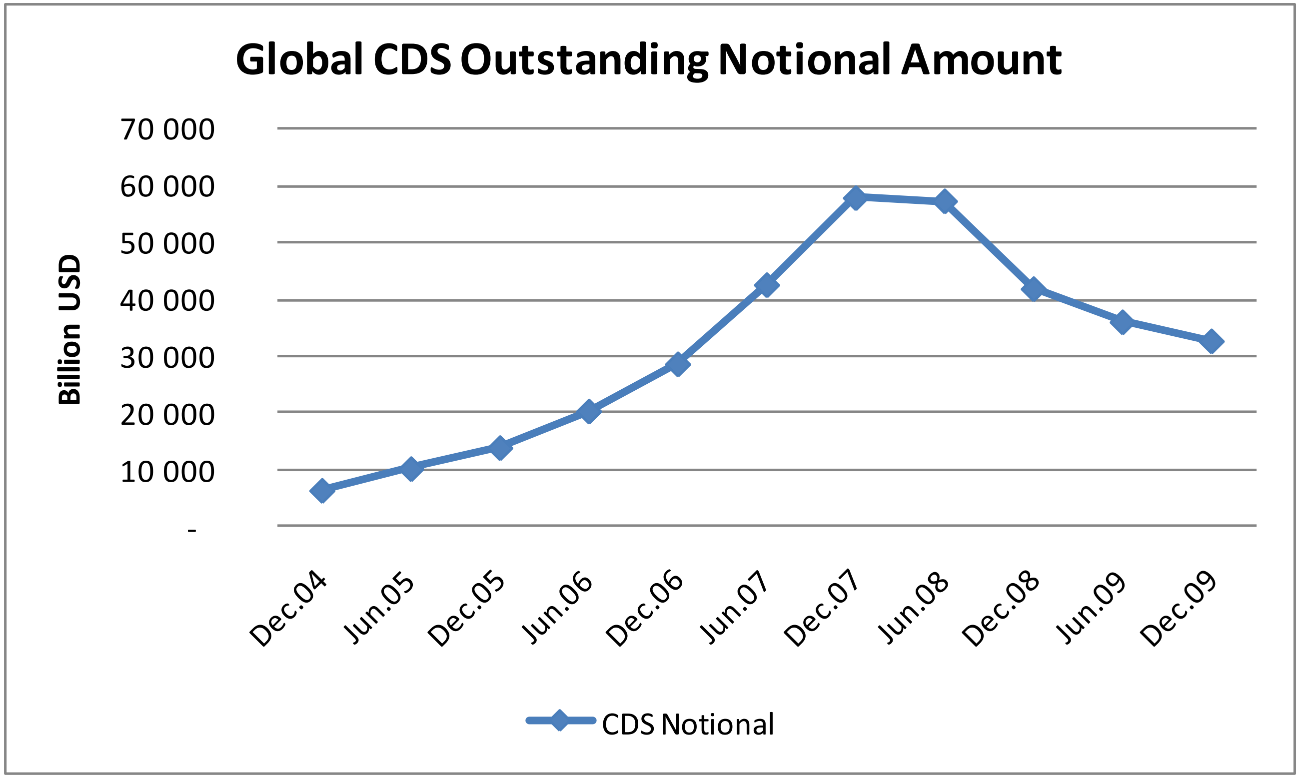
Content

- ✓ Introduction
- ✓ Credit Risk Management
- ✓ Rating and Scoring Systems
- ✓ Portfolio Credit Risk Modeling
- ➢ Credit Derivatives

Credit Derivatives

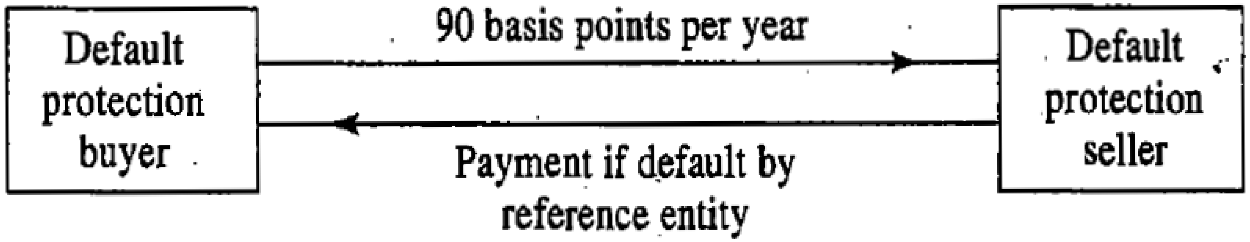
- Payoff depends on creditworthiness of one or more subjects
- Single name or multi-name
- Credit Default Swaps, Total Return Swaps, Asset Backed Securities, Collateralized Debt Obligations
- Banks – typical buyers of credit protection, insurance companies – sellers

Credit Default Swaps



Credit Default Swaps

- CDS spread usually paid in arrears quarterly until default
- Notional, maturity, definition of default
- Reference entity (single name)
- Physical settlement – protection buyer has the right to sell bonds (CTD)
- Cash settlement – calculation agent, or binary
- Can be used to hedge corresponding bonds



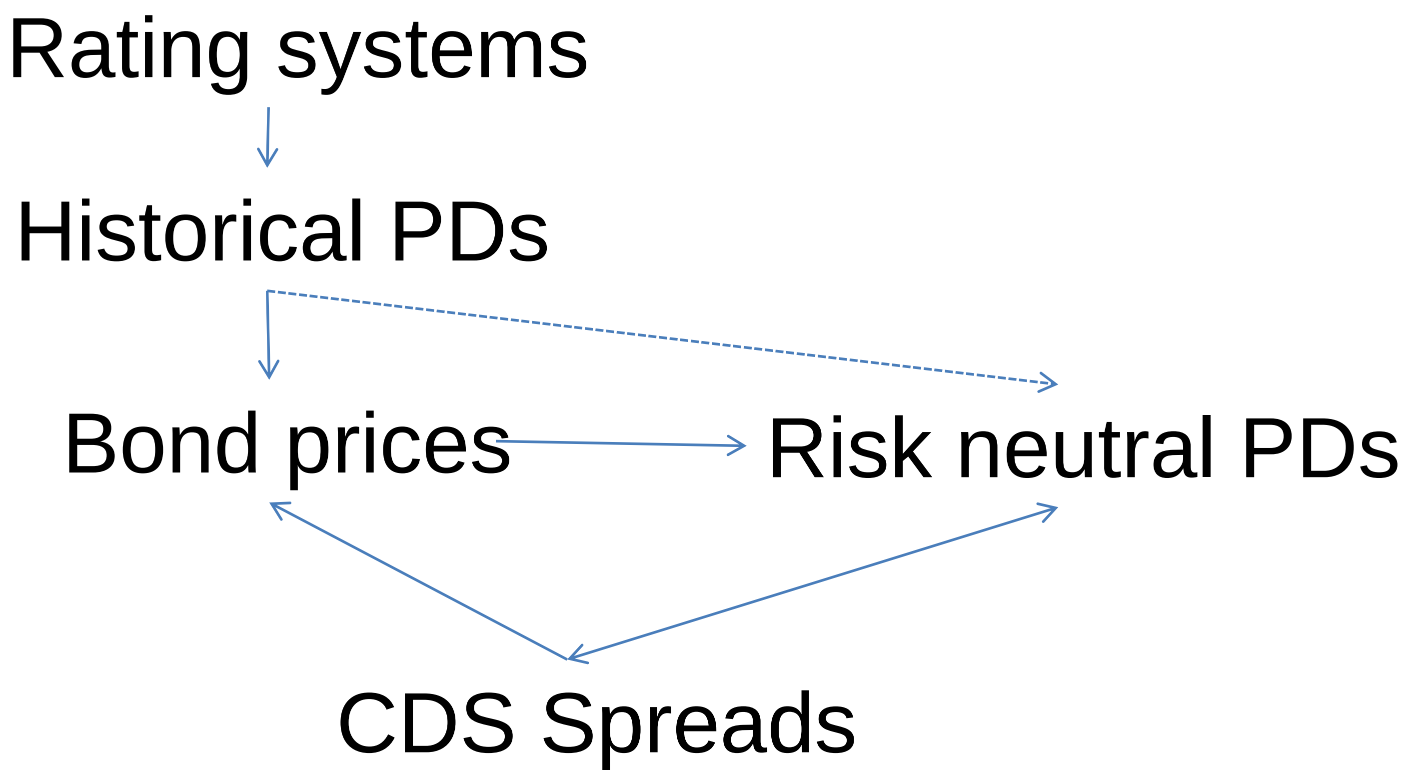
Valuation of CDS

- Requires risk neutral probabilities of default for all relevant maturities
- Market value of a CDS position is then based on the general formula

$$MV = E_Q[\text{discounted cash flow}] = \sum_{i=1}^{n} e^{-r_i T_i} E_Q[\text{cash flow}_i]$$

- Market equilibrium CDS spread is the spread that makes *MV*=0

Estimating Default Probabilities



Credit Indices

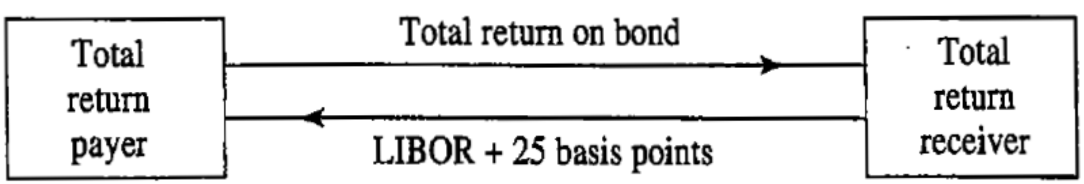
- In principle averages of single name CDS spreads for a list of companies
- In practice traded multi-name CDS
- CDX NA IG – 125 investment grade companies in N.America
- iTraxx Europe - 125 investment grade European companies
- Standardized payment dates, maturities (3,5,7,10), and even coupons – market value initial settlement

CDS Forwards and Options

- Defined similarly to forwards and options on other assets or contracts, e.g. IRS
- Valuation of forwards can be done just with the term structure of risk neutral probabilities
- But valuation of options requires a stochastic modeling of probabilities of default (or intensities – hazard rates)

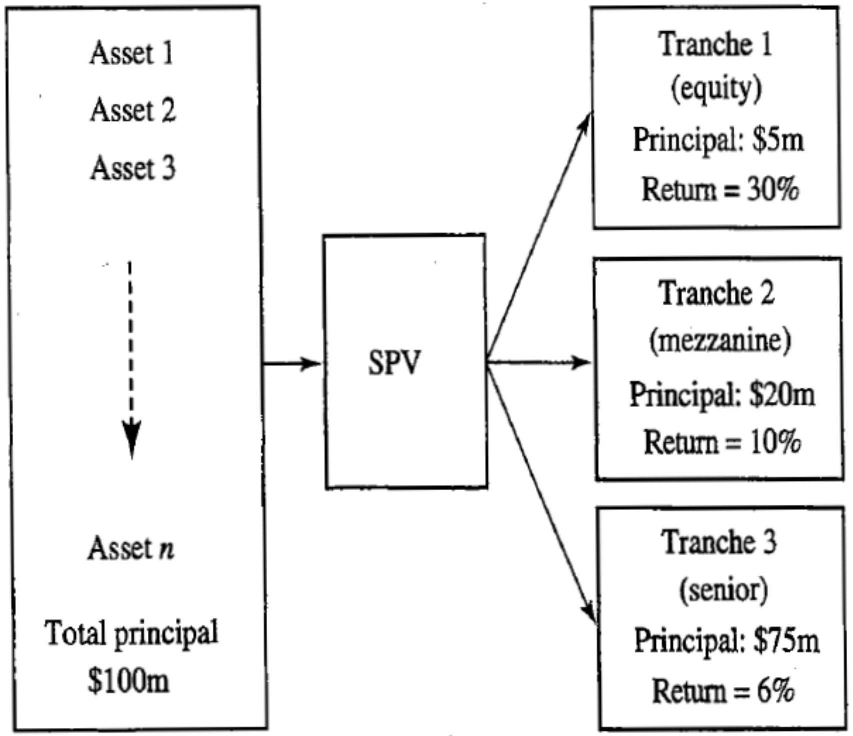
Basket CDS and Total Return Swaps

- Many different types of basket CDS: add-up, first-to-default, k-th to default … requires credit correlation modeling
- The idea of Total Return Swap (compare to Asset Swaps) is to swap total return of a bond (or portfolio) including credit losses for Libor + spread on the principal (the spread covers the receivers risk of default!)

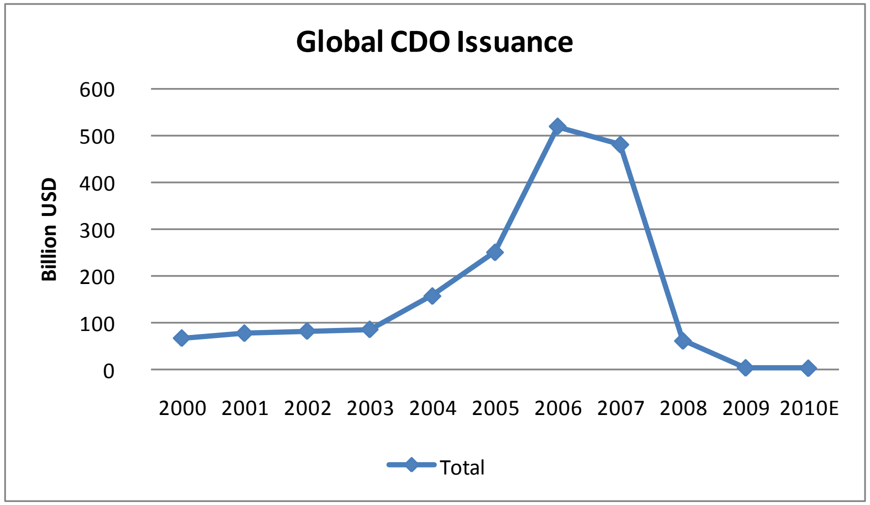


Asset Backed Securities (CDO,..)

- Allow to create AAA bonds from a portfolio of poor assets



CDO market



Year	High Yield Bonds	High Yield Loans	Investment Grade Bonds	Mixed Collateral	Other	Other Swaps	Structured Finance	Total
2000	11 321	22 715	29 892	2 090	932		1 038	**67 988**
2001	13 434	27 368	31 959	2 194	2 705		794	**78 454**
2002	2 401	30 388	21 453	1 915	9 418		17 499	**83 074**
2003	10 091	22 584	11 770	22	6 947	110	35 106	**86 630**
2004	8 019	32 192	11 606	1 095	14 873	6 775	83 262	**157 821**
2005	1 413	69 441	3 878	893	15 811	2 257	157 572	**251 265**
2006	941	171 906	24 865	20	14 447	762	307 705	**520 645**
2007	2 151	138 827	78 571		1 722	1 147	259 184	**481 601**
2008		27 489	15 955				18 442	**61 887**
2009		2 033	1 972				331	**4 336**

Cash versus Synthetic CDOs

- Synthetic CDOs are created using CDS

BY ISSUANCE TYPE ($MM)

	TOTAL ISSUANCE	Cash Flow and Hybrid[2]	Synthetic Funded[3]	Market Value[4]	Arbitrage[5]	Balance Sheet[6]	Long Term[7]	Short Term[8]
2004-Q1	24,982.5	18,807.8	6,174.7	0.0	23,157.5	1,825.0	20,495.1	4,487.4
2004-Q2	42,861.6	25,786.7	17,074.9	0.0	39,715.5	3,146.1	29,611.4	13,250.2
2004-Q3	42,086.6	36,106.9	5,329.7	650.0	38,207.7	3,878.8	34,023.9	8,062.7
2004-Q4	47,487.8	38,829.9	8,657.9	0.0	45,917.8	1,569.9	38,771.4	8,716.4
2004 TOTAL	**157,418.5**	**119,531.3**	**37,237.2**	**650.0**	**146,998.5**	**10,419.8**	**122,901.8**	**34,516.7**
2005-Q1	49,610.2	40,843.9	8,766.3	0.0	43,758.8	5,851.4	45,175.2	4,435.0
2005-Q2	71,450.5	49,524.6	21,695.9	230.0	62,050.5	9,400.0	65,043.6	6,406.9
2005-Q3	52,007.2	44,253.1	7,754.1	0.0	49,636.7	2,370.5	48,656.3	3,350.9
2005-Q4	98,735.4	71,604.3	26,741.1	390.0	71,957.6	26,777.8	88,763.5	9,971.9
2005 TOTAL	**271,803.3**	**206,225.9**	**64,957.4**	**620.0**	**227,403.6**	**44,399.7**	**247,638.6**	**24,164.7**
2006-Q1	108,012.7	83,790.1	24.222.6	0.0	101,153.6	6,859.1	104,084.0	3,928.7
2006-Q2	124,977.9	97,260.3	24,808.4	2,909.2	102,564.6	22,413.3	119,986.1	4,991.8
2006-Q3	138,628.7	102,167.4	14,703.8	21,757.5	125,945.2	12,683.5	135.928.5	2,700.2
2006-Q4	180,090.3	131,525.1	25,307.9	23,257.3	142,534.3	37,556.0	180,090.3	0.0
2006 TOTAL	**551,709.6**	**414,742.9**	**89,042.7**	**47,924.0**	**472,197.7**	**79,511.9**	**540,088.9**	**11,620.7**
2007-Q1	186,467.6	140,319.1	27,426.2	18,722.3	156,792.0	29,675.6	181,341.2	5,126.4
2007-Q2	175,939.4	135,021.4	8,403.0	32,515.0	153.385.4	22,554.0	167,459.2	8,480.2
2007-Q3	93,063.6	56,053.3	5,198.9	31,811.4	86,331.4	6,732.2	90,710.0	2,353.6
2007-Q4	47,508.2	31,257.9	5,202.3	11,048.0	39,593.7	7,914.5	47,508.2	0.0
2007 TOTAL	**502,978.8**	**362,651.7**	**46,230.4**	**94,096.7**	**436,102.5**	**66,876.3**	**487,018.6**	**15,960.2**
2008-Q1**	19,470.7	11,930.1	513.7	7,026.9	18,111.8	1,358.9	19,470.7	0.0
2008-Q2	17,336.7	14,260.4	698.5	2,377.8	10,743.7	6,593.0	17,336.7	0.0
2008 YTD TOTAL	**36,807.4**	**26,190.5**	**1,212.2**	**9,404.7**	**28,855.5**	**7,951.9**	**36,807.4**	**0.0**

Single Tranche Trading

- Synthetic CDO tranches based on CDX or iTraxx

Table 23.6 Five-year CDX NA IG and iTraxx Europe tranches on March 28, 2007. Quotes are 30/360 in basis points except for 0%–3% tranche, where the quote indicates the percent of the tranche principal that must be paid up front in addition to 500 basis points per year.

CDX NA IG						
Tranche	0–3%	3–7%	7–10%	10–15%	15–30%	30–100%
Quote	26.85%	103.8	20.3	10.3	4.3	2.0
iTraxx Europe						
Tranche	0–3%	3–6%	6–9%	9–12%	12–22%	22–100%
Quote	11.25%	57.7	14.4	6.4	2.6	1.2

John Hull, Option, Futures, and Other Derivatives, 7th Edition

Valuation of CDOs

- Sources of uncertainty: times of default of individual obligor and the recovery rates (assumed deterministic in a simplified approach)
- Everything else depends on the Waterfall rules (but in practice often very complex to implement precisely)

Valuation of CDOs

- Monte Carlo simulation approach:
- In one run simulate the times to default of individual obligors in the portfolio using risk neutral probabilities and appropriate correlation structure
- Generate the overall cash flow (interest and principal payments) and the cash flows to individual tranches
- Calculate for each tranche the mean (expected value) of the discounted cash flows

Gaussian Copula of Time to Default

- Guassian Copula is the approach when correlation is modeled on the standard normal transformation of the time to default

$$X_j = N^{-1}(Q_j(T_j))$$

$$X_j = \sqrt{\rho} \cdot M + \sqrt{1-\rho} \cdot Z_j$$

- The single factor approach can be used to obtain an analytical valuation
- Generally used also in simulations

Implied Correlation

- Correlations implied by market quotes based on the standard one factor model (similarly to implied volatility)

Table 23.8 Implied correlations for 5-year iTraxx Europe tranches on March 28, 2007.

Compound correlations					
Tranche	0–3%	3–6%	6–9%	9–12%	12–22%
Quote	18.3%	9.3%	14.3%	18.2%	24.1%
Base correlations					
Tranche	0–3%	0–6%	0–9%	0–12%	0–22%
Quote	18.3%	27.3%	34.9%	41.4%	58.1%

John Hull, Option, Futures, and Other Derivatives, 7th Edition

Alternative Models

- The correlations are uncertain!
- The Gaussian correlations may go up if there is a turmoil on the market!
- Alternative copulas: Student t copula, Clayton copula, Archimedean copula, Marshall-Olkin copula
- Random factor loading $x_i = a(F)F + \sqrt{1 - a(F)}\, Z_i$
- Dynamic models – stochastic modeling of portfolio loss over time – structural (assets), reduced form (hazard rates), top down models (total loss)