Operational Research

Theory and Applications

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ALMA MATER STUDIORUM
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Summary

- What is O.R.?
- The birth of O.R. during WW2
- O.R. models
- The role of O.R. in business decisions
- Famous problems solved by O.R.

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What is Operational Research?

What is operational research?

"In a nutshell, operational research (O.R.) is the discipline of applying appropriate analytical methods to help make better decisions."

— The O.R. Society

What is operational research?

"A method of mathematically based analysis for providing a quantitive basis for management decisions."

— The Oxford Dictionary

What is operational research?

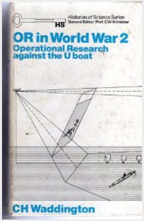
O.R. is closely related with the concept of optimisation:

"The process of finding the best possible choice out of a set of alternatives."

— The OR Society

The birth of O.R. during WW2

The birth of O.R.



As most technologies, O.R. was born for military applications, in England, during WW2.

In then spread to the United States and other countries.

After the war, it started finding civilian applications in the most diverse fields.

Optimisation: a WW2 example

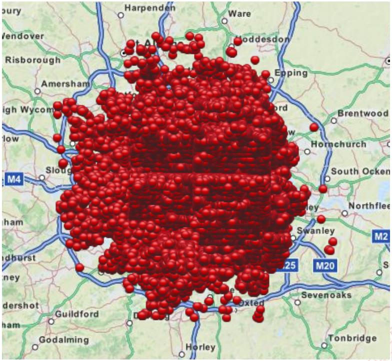
Bombs dropped by Germany on London during 7th September 1940.

Optimisation: a WW2 example

Bombs dropped by Germany on London during the subsequent week.

Optimisation: a WW2 example

Bombs dropped by Germany on London during the subsequent 8 months.



Optimisation: a WW2 example

Anti-bomb shelters didn't prove very effective and people started using the subway stations as shelters.

Should the government approve or recommend such actions?

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Optimisation: a WW2 example

Anti-bomb shelters didn't prove very effective and people started using the subway stations as shelters.

Should the government approve or recommend such actions? pros cons

Optimisation: a WW2 example

What is the minimum depth of a tunnel to allow to take refuge in it? We want to:

minimise	the number of human losses from air raids
such that	all people in London have a fair chance to find refuge close to them

Optimisation: a WW2 example

Decisions were taken according to experience, intuition, common sense.

Some of these decisions can be improved via a **scientific approach**.

The first group of O.R. scientist was heavily **multidisciplinary**.

Optimisation: another WW2 example

The british have to protect n strategic objectives using m radars, where n is much bigger than m.

Roughly a radar can patrol a circular area around its position.

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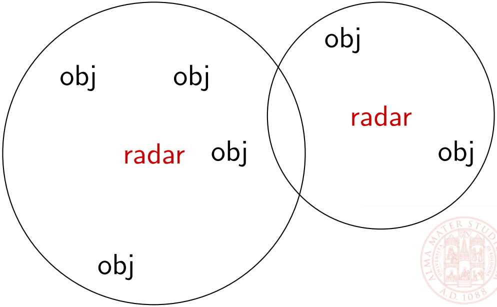

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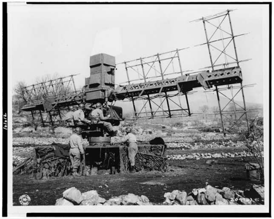
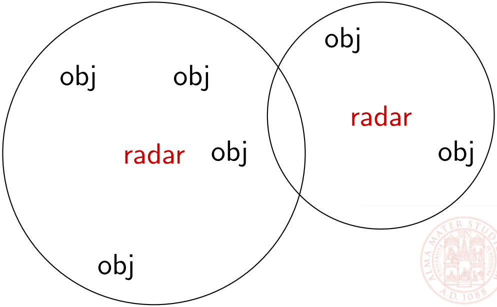




Optimisation: another WW2 example

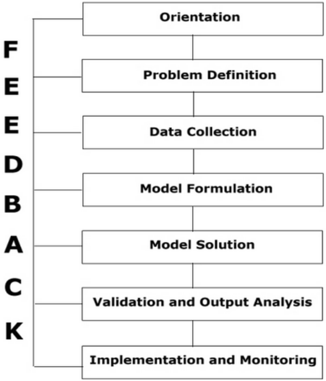
Every strategic objective $i = 1, \dots, n$ has a certain strategic value v_i.

We want to find the position in which to place the radars:

maximise	the total value of protected objectives $\sum_{i=1}^{n} v_i$
such that	we use at most m radars (the number of available radars)

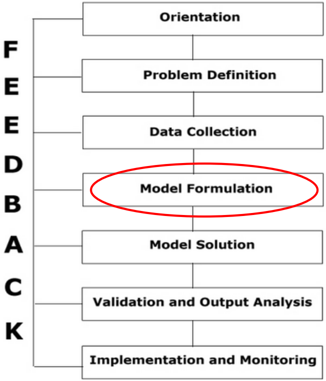
O.R. models

The workflow of the O.R. scientist



It looks a lot like the **scientific method**, doesn't it?

The workflow of the O.R. scientist



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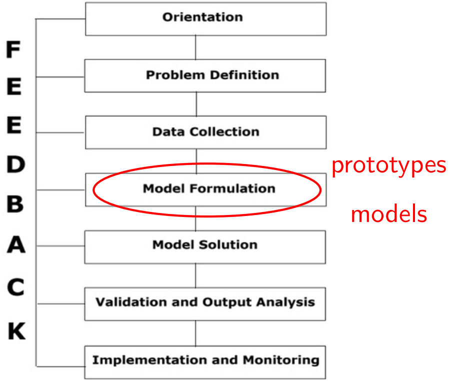

It looks a lot like the **scientific method**, doesn't it?

Models

Models constitute a flexible, cheap and accurate way to represent reality and to run experiments without having to physically implement them.

Example: an ice-cream factory produces 2 types of ice cream: "**chocolate dream**" and "**voluptuous white**".

Models: an example

One hour worth of production of "chocolate dream" requires 50 liters of milk and 20 kilograms of chocolate powder.

One hour worth of production of "voluptuous white" requres 70 liters of milk, 5 kilograms of chocolate powder and 1 kilogram of vanilla.

The first profits 500$ per hour and the second profits 400$ per hour.

Models: an example

Ice cream	Milk	Chocolate	Vanilla	Profit
Chocolate dream	50	20	0	500
Voluptuous white	70	5	1	400

The factory can only get 800 liters of milk, 200 kilograms of chocolate and 14 kilograms of vanilla per day.

The factory can work up to 18 hours per day.

Models: an example

We are required to build a **model** to decide how many hours per day to produce “chocolate dream” vs. “voluptuous white”.

Of course we want to maximise profits, but we also have to satisfy certain constraints: maximum available quantity of ingredients per day and maximum number of working hours per day.

Models: an example

We denote with x_c the number of hours per day during which the factory should produce "chocolate dream" and with x_v the number of hours during which it should produce "voluptuous white".

These are our **unknowns**, or variables. In our case, they can only be non-negative numbers.

Models: an example

Since our two different kinds of ice cream profit 500$ and 400$ per hour, in order to maximise total profit, we have to maximise the quantity:

$$500 \cdot x_{\mathsf{c}} + 400 \cdot x_{\mathsf{v}}$$

Clearly only producing "chocolate dream" 18 hours per day would be the most convenient choice. Unfortunately, we don't have enough raw materials to do that.

Models: an example

We have to take into account our constraints:

- We can't work more than 18 hours per day overall (and we can only produce one ice cream type at a time):

$$x_{\mathsf{c}} + x_{\mathsf{v}} \leq 18$$

Models: an example

We have to take into account our constraints:

- We can't work more than 18 hours per day overall (and we can only produce one ice cream type at a time):

$$x_{\mathsf{c}} + x_{\mathsf{v}} \leq 18$$

- We can't use more raw material than available per day:

$$\begin{array}{rlll} \text{milk:} & 50 \cdot x_{\mathsf{c}} & + 70 \cdot x_{\mathsf{v}} & \leq 800 \\ \text{chocolate:} & 20 \cdot x_{\mathsf{c}} & + 5 \cdot x_{\mathsf{v}} & \leq 200 \\ \text{vanilla:} & & x_{\mathsf{v}} & \leq 14 \end{array}$$

Models: an example

Finally, our **model** looks like this:

$$
\begin{array}{rllr}
\text{maximise} & 500 \cdot x_{\mathsf{c}} & + 400 \cdot x_{\mathsf{v}} & \\
\text{such that} & 50 \cdot x_{\mathsf{c}} & + 70 \cdot x_{\mathsf{v}} & \leq 800 \\
 & 20 \cdot x_{\mathsf{c}} & + 5 \cdot x_{\mathsf{v}} & \leq 200 \\
 & & \quad x_{\mathsf{v}} & \leq 14 \\
 & x_{\mathsf{c}} & + x_{\mathsf{v}} & \leq 18 \\
 & x_{\mathsf{c}} & \ , x_{\mathsf{v}} & \geq 0
\end{array}
$$

Models: an example

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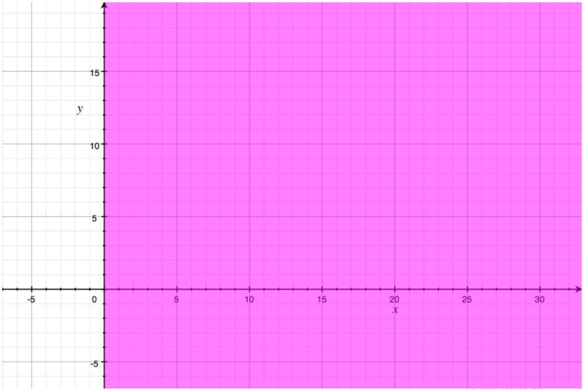
 & x_{\mathsf{c}} & \ , x_{\mathsf{v}} & \geq 0
\end{array}
$$

Let's have a look at a graphical representation of this model.

Models: an example

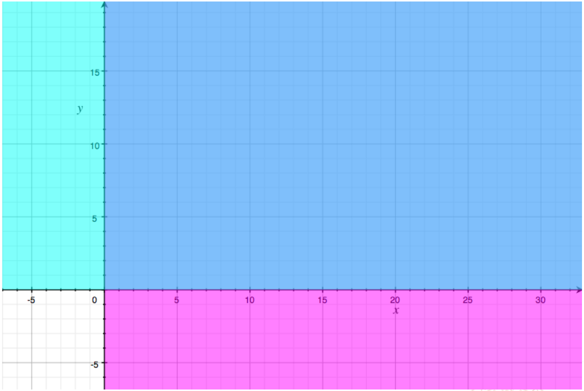
Constraint: $x_c \geq 0$



Horizontal axis is x_c, vertical axis is x_v.

Models: an example

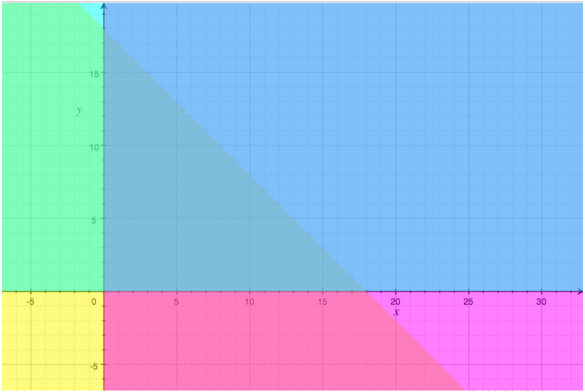
New constraint: $x_{\mathsf{v}} \geq 0$



Horizontal axis is x_{c}, vertical axis is x_{v}.

Models: an example

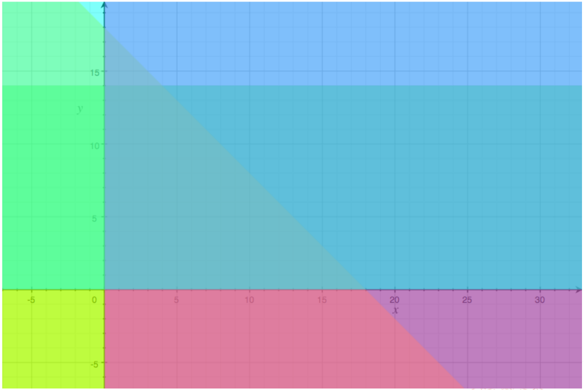
New constraint: $x_c + x_v \leq 18$



Horizontal axis is x_c, vertical axis is x_v.

Models: an example

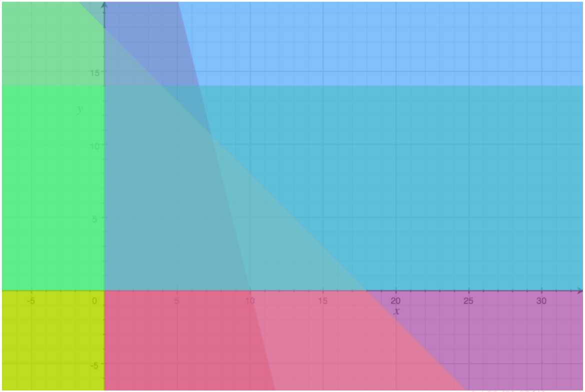
New constraint: $x_{\mathsf{v}} \leq 14$



Horizontal axis is x_{c}, vertical axis is x_{v}.

Models: an example

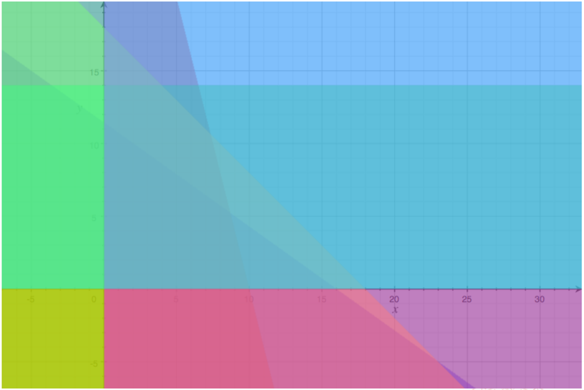
New constraint: $20x_{\mathsf{c}} + 5x_{\mathsf{v}} \leq 200$



Horizontal axis is x_{c}, vertical axis is x_{v}.

Models: an example

New constraint: $50x_{\mathsf{c}} + 70x_{\mathsf{v}} \leq 800$



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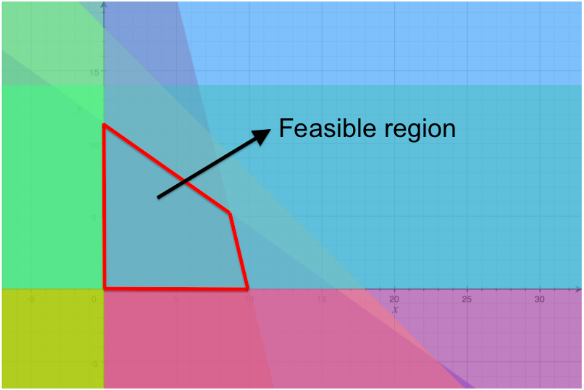
Models: an example



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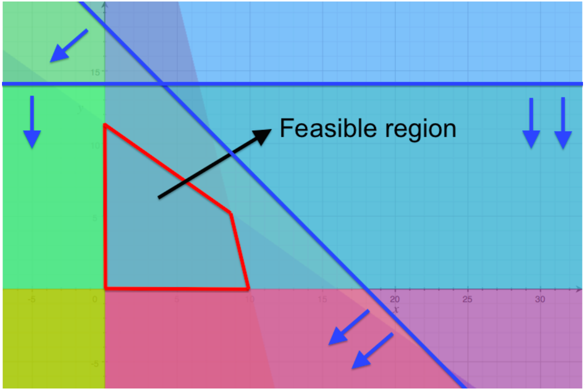


Horizontal axis is x_c, vertical axis is x_v.

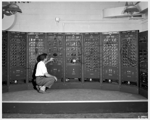
Applications of O.R. to business

O.R. in business

After WW2 the potential of application of O.R. in civilian settings became clear.

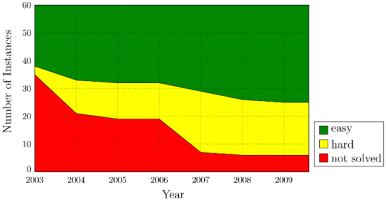
O.R. research groups spread from Great Britain to the rest of Europe, to the United States and eventually allover the world.

O.R. in business



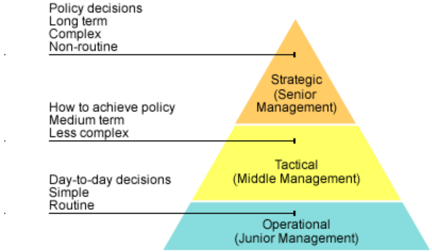
In the beginning O.R. proposed methods and algorithms to efficiently solve complex problems... by hand!

Eventually computers became more powerful and affordable revolutionising the way we do science and business!



O.R. in business

Where does O.R. intervene in the business decision-making process?



O.R. in business

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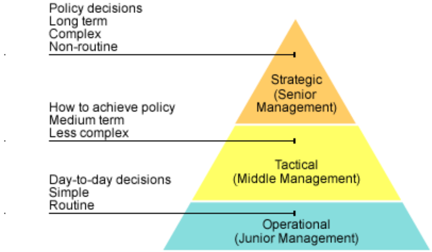


Answer: everywhere!

O.R. in business

Example: Mærsk Line

MAERSK LINE

Strategic (years): fleet acquisition, network design

Tactical (months): routes design, fleet deployment

Operational (days): stowage design, disruption management

O.R. in business

Example: Trenitalia/RFI

Strategic (years): infrastructure, trains acquisition

Tactical (months): train schedules, personnel schedules

Operational (days): personnel rostering, rescheduling and rerouting

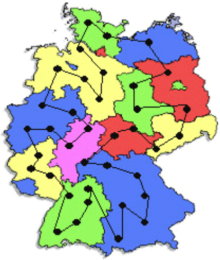
Famous problems solved by O.R.

- Vehicle routing problem
- Facility location problem
- Production planning: lot-sizing problem
- Knapsack problem
- Scheduling problem
- Applications to finance and accounting

Vehicle routing problem

It was the first application of O.R. to a real-life civilian problem (Dantzig and Ramser, 1959).

Simplest case: Travelling Salesman Problem (TSP). A salesman has to visit a certain number of cities and wants to save on fuel as much as possible.



Vehicle routing problem

In the TSP we have to find a visit order that minimises the total distance travelled, while visiting all cities. In other cases we might want to minimise the time travelled. The two things don't always coincide!

There are many variants of the TSP:

- Symmetric vs. asymmetric
- With time windows
- With pick-ups and deliveries (and vehicle capacity)

Vehicle routing problem

Remember: the TSP is a model. As such, it can be used to solve a lot of different problems, even those that don't involve salesmen at all!

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- Garbage collection

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- Garbage collection
- Postal delivery

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- Garbage collection
- Postal delivery
- Supermarkets distribution

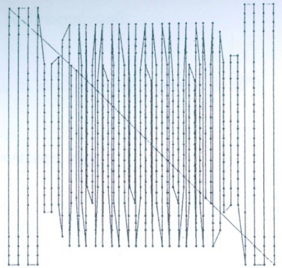
Vehicle routing problem

Remember: the TSP is a model. As such, it can be used to solve a lot of different problems, even those that don't involve salesmen at all!

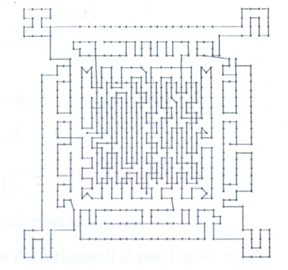
- Garbage collection
- Postal delivery
- Supermarkets distribution
- And even...

Vehicle routing problem

...perforation of ceramics supports for multi-chip modules:



Before an O.R. scientist worked on the problem



After applying O.R.: around 30% time reduction

Vehicle routing problem

The vehicle routing problem (VRP) is basically the multi-vehicle version of the TSP.

Given a fleet of vehicles based at one or more depots, find a route for each vehicle such that all customers are visited and the total travelled distance is minimal.

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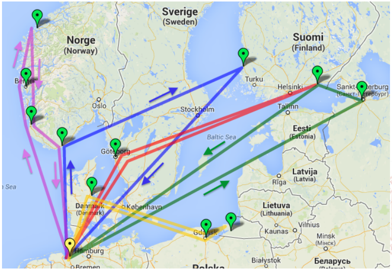


Vehicle routing problem

Again many variants exist:

- Symmetric vs. asymmetric; with time windows; with pick-ups and deliveries
- With split pick-ups and deliveries
- With back-hauls
- Dial-a-ride problems
- Emergency/humanitarian scenarios
- Robust VRP
- Multi-echelon routing

Vehicle routing problem

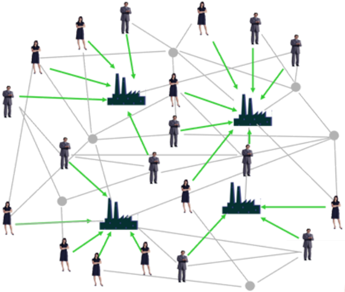
Strategic questions: Is our fleet over-/under-sized? Is it enough to expand to new markets?

Tactical questions: How many vehicles should we deploy at each depot? Should we increase/decrease their capacity?

Operational questions: Which customers will I visit today? In which order? Taking which route?

Facility location problem

Given a set of customers and a set of possible facilities locations, which facilities to open in order to minimise the cost of servicing all customers? **Notice:** opening a facility incurs in a cost!



Facility location problem

Facilities can be **factories**, **warehouses**, etc.

Cost of servicing a customer can depend on the distance or any other factor.

Fixed cost when opening a facility... without this one the optimal solution would probably be: open all possible facilities!

Facility location problem

Again this model can be applied to different scenarios:

Facility location problem

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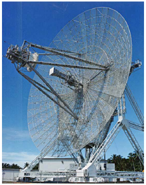
- Facilities are **ambulance posts** and customers are neighbourhoods, blocks, etc.

Facility location problem

Again this model can be applied to different scenarios:

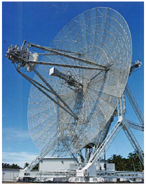
- Facilities are **ambulance posts** and customers are neighbourhoods, blocks, etc.
- Facilities are **radars** and customers are strategic objectives (as we have seen!)



Facility location problem

Again this model can be applied to different scenarios:

- Facilities are **ambulance posts** and customers are neighbourhoods, blocks, etc.
- Facilities are **radars** and customers are strategic objectives (as we have seen!)
- Facilities are **power plants** and customers are energy distribution substations



Facility location problem

Strategic questions: How many facilities should we open? Where?

Tactical questions: Which customers will be served by each facility?

Production planning - Lot sizing

A manufacturing company is given the forecast of the orders it will receive in the upcoming days/weeks/months (time interval).

Production planning - Lot sizing

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At each time interval it can start production (paying a fixed start-up cost) and decide how much to produce, up to a certain maximum.

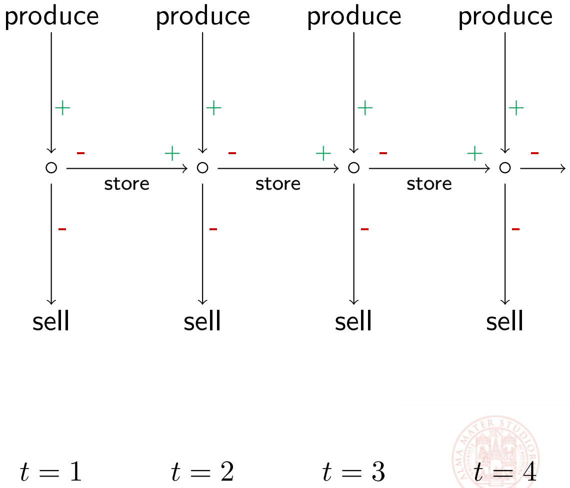
Production planning - Lot sizing

A manufacturing company is given the forecast of the orders it will receive in the upcoming days/weeks/months (time interval).

At each time interval it can start production (paying a fixed start-up cost) and decide how much to produce, up to a certain maximum.

If it produced more than it needs to deliver, it can store goods in a warehouse (paying a storage cost).

Production planning - Lot sizing



Production planning - Lot sizing

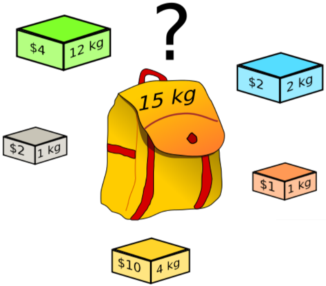
Strategic questions: Do we need to increase our mfg capacity? Do we need to shorten our supply chain? Do we need additional warehouse space?

Tactical questions: How much material should we buy? How many orders can we take?

Operational questions: How much to produce/store today?

Knapsack problem

Given a knapsack with a certain capacity (e.g. maximum weight it can carry) and many elements, each with a certain weight and value, find a combination of these elements that fits in the knapsack and maximises the total value.

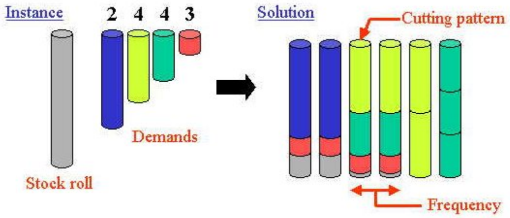


Knapsack problem

Example: a thief in an apartment!

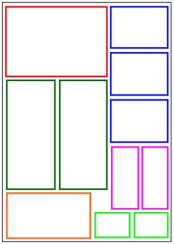
Knapsack problem

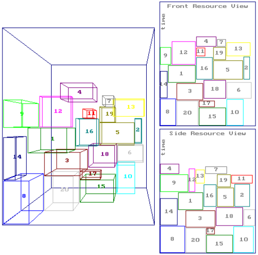
Example: the cutting stock problem (multi-knapsack problem).



Knapsack problem

The previous **example** can be extended to the 2D and 3D cases.

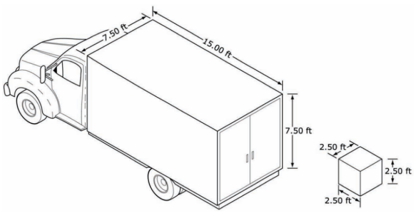


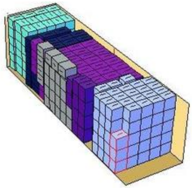


If we forget about the cuts, this becomes a packing problem!

Knapsack problem → packing problem

For example, if we want to maximise space utilisation in a lorry, or a train car, or a ship!





Knapsack problem → packing problem

Additional constraints:

- **Weight distribution**, e.g. on a ship the weight must be equally distributed
- **Unload order constraints**, e.g. don't put a box to be unloaded early below another to be unloaded later
- **Neighbour constraints**, e.g. dangerous goods that can't stay close to each other

Knapsack problem

More examples:

- Job scheduling in computer clusters (multi-knapsack) or in manufacturing machines (weight: expected running time, prize: priority).
- Stock portfolio composition in financial trading (weight: cost, prize: expected profit). **Stochastic optimisation**.

Knapsack problem

Using the knapsack problem to cheat the system: the secret ballot problem.

Knapsack problem

Using the knapsack problem to cheat the system: the secret ballot problem.

	p_i	a	b	c	d	e	f	
1	14.3%							
2	13.2%							
3	12.4%							
4	8.4%							
5	7.8%							
6	6.2%							
7	5.7%							
8	5.5%							
9	4.5%							
10	4.2%							
11	3.6%							
12	3.1%							
13	2.7%							
14	2.4%							
15	1.5%							
16	1.4%							
17	1.3%							
18	1.1%							
19	0.4%							
20	0.3%							
		35.9%	11.1%	17.4%	17.3%	13.8%	4.5%	v_j

Knapsack problem

Can you figure out who voted whom?

Knapsack problem

Can you figure out who voted whom?

		a	b	c	d	e	f	
1	14.3%	1						1
2	13.2%	1						1
3	12.4%				1			1
4	8.4%			1				1
5	7.8%					1		1
6	6.2%		1					1
7	5.7%	1						1
8	5.5%			1				1
9	4.5%				1			1
10	4.2%					1		1
11	3.6%		1					1
12	3.1%						1	1
13	2.7%	1						1
14	2.4%			1				1
15	1.5%					1		1
16	1.4%						1	1
17	1.3%		1					1
18	1.1%			1				1
19	0.4%				1			1
20	0.3%					1		1
		35.9%	11.1%	17.4%	17.3%	13.8%	4.5%	

Scheduling problems

The scheduling problem consists in creating work schedules for employees and assigning them to specific tasks, so that all the tasks are covered by some employee.

Usually the **objective** is to minimise the number of employees needed, but sometimes it can be to meet a certain quality of service, etc.

Constraints include working regulations, employee specialisation, employee preferences, shifts, etc.

Scheduling problems

Again, O.R. flexibility comes into play: this model can be used for many different things.

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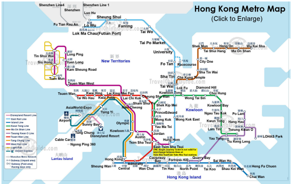
For example, to schedule airplanes to available runways such that no two airplanes use the same runway at the same time and all airplanes land before they run out of fuel!

Here: worker → runway, job → airplane.

Scheduling problems

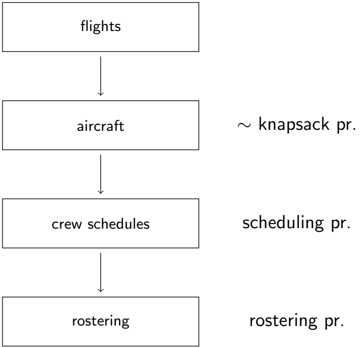
The scheduling problem is often a dynamic problem, because not all jobs to schedule might be known in advance.

Example: MTR (**Hong Kong Metro**) schedules engineers on their lines, with over 200 different rules. They managed to give their engineers 30 additional minutes on average to complete a maintenance task and saved $800,000 per year.



Scheduling problems

A more complicated example comes from Air Canada:



Scheduling problems

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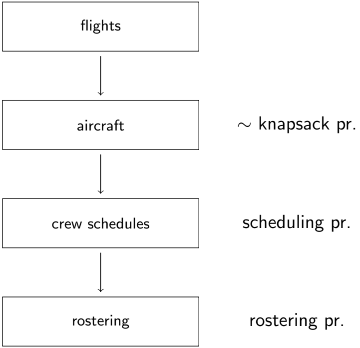

The solution implemented for Air Canada allowed savings for 7.8%. After negotiations with unions, they closed a deal allowing 2.03% in savings. For an airline this is **huge**!

Summary

Summary

Operational Research: analytical tools to make better decisions

We saw example problems for which O.R. can provide not only a **good** solution, but the **best** (optimal) one.

In general, if there's quantitative data and some operations, O.R. can improve the operations.

Summary

We saw **applications** mainly in the fields of manufacturing and logistics, but O.R. has been used everywhere:

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- Healthcare: Nurses and doctors scheduling, operating theatres allocation, fitness plans optimisation, etc.
- Artificial Intelligence: Robotics, Data Mining, Computer Vision, etc.

More questions?

Tomorrow, same time (18:00 - 20:00) and same place

1-to-1 consulting with entrepreneurs, engineers, researchers, students