Masters in
DEVELOPMENT FINANCE

*Financial Risk Management**

Prescribed Textbook:
Lam, J. (2003), *Enterprise Risk Management: From Incentives to Controls*, John Wiley & Sons, USA.

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CHAPTER 1: INTRODUCTION

1.1 Definitions and Concepts

1.1.1 Definitions of 'Risk'[1]

Historically the term *risk* had different meanings to those used today. During the middle ages, the Arabian term *risicum* was used in the context of good or bad fortune associated with loss or damage arising from sea trade.[2] The concept of marine risk, termed *rischio* and *riezgo*, was then adopted in 16th Century Europe as European merchants began to trade with Middle Eastern and North African Arab traders. However, the term was still associated with good or bad fortune rather than being a quantifiable concept because the mathematical tools needed to measure risk were yet to be invented.

In the 17th century the concept of risk shed the good fortune side of unforeseen events and focussed on the downside exclusively, becoming the modern concept of risk still in use today.[3] The early spelling of the term in English is in the Oxford English Dictionary, which spelt the word as *risque* in the 1621 edition before becoming *risk* from 1655. During this time the term was defined as exposure to the possibility of loss, injury, or other adverse or unwelcome circumstance; a chance or situation involving such a possibility.[4]

In a modern context, ISO 31000 Guide 73 defines risk as *the effect of uncertainty on objectives*, which can include uncertainties arising from events that may or may not happen or from a lack of information. In statistics, risk is typically modelled as the expected value arising from the probability of an undesirable occurrence. In decision theory, risk is typically modelled as the expected value of a loss function arising from the decision rule used to make the decision in the face of uncertainty.

1.1.2 Risk versus Uncertainty

Frank Knight is widely regarded as having established the distinction between risk and uncertainty in his book entitled *Risk, Uncertainty, and Profit* (1921: pg. 19).[5]

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1 Bernstein, P.L., (1998), *Against the Gods*, John Wiley & Sons, New York.
2 Luhmann, N. (1996:4), "Modern Society Shocked by its Risks," University of Hong Kong, *Department of Sociology Occasional Papers 17.*
3 Franklin, J. (2001), *The Science of Conjecture: Evidence and Probability Before Pascal*, Johns Hopkins University Press, Baltimore.
4 Oxford English Dictionary.
5 Knight, F.H. (1921), *Risk, Uncertainty and Profit*, Schaffner, H. (ed.), Marx Prize Essays, No. 31, Houghton Mifflin, New York.

> *Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated. The term 'risk,' as loosely used in everyday speech and in economic discussion, really covers two things which, functionally at least, in their causal relations to the phenomena of economic organization, are categorically different. [...] The essential fact is that 'risk' means in some cases a quantity susceptible to measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomenon depending on which of the two is really present and operating. [...] It will appear that a measurable uncertainty, or 'risk' proper, as we shall use the term, is so far different from an un-measurable one that it is not in effect an uncertainty at all. We [...] accordingly restrict the term 'uncertainty' to cases of the non-quantitative type.*

Thus according to Knight the primary difference between risk and uncertainty is that risk can be measured while uncertainty cannot.

1.2 Financial Risk

Recommended Reading: Textbook Chapters 1 – 3, 16 and 18

1.2.1 Concepts and Definitions

In finance the concept of risk has a variety of definitions, but generally risk can be defined as the level of regret or disappointment. More specifically, risk is the probability that the return of an investment will be different to that expected while financial risk may be defined as the risk associated with financing. Hence, financial risk can be defined as the unexpected variability or volatility of returns.

However, in contrast to the classic definitions of risk, financial risk can include both up-side and down-side risk.[6] *Downside risk* occurs when the actual return is less than the expected return while *up-side risk* occurs when the actual return is more than the expected return. A fundamental concept in the modern portfolio theory of finance is the relationship between risk and return whereby the greater the return being sought, the greater the risk being assumed.

1.2.2 Financial Risk Management

Financial risk management involves the use of financial instruments to manage a firm's exposure to financial risk. A key activity associated with financial risk management is hedging. The *hedging irrelevance proposition* states that in a perfect market, a company cannot create value by hedging a risk

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[6] Damodaran, A. (2003), *Investment Philosophies: Successful Investment Philosophies and the Greatest Investors Who Made Them Work*, Wiley, New York.

where the price of bearing that risk is the same as the price outside the company. Hence, this implies that financial risk management will only create value when the company can manage risks more cost effectively than the shareholders can. Thus market risks and firm-specific risks are best mitigated by financial risk management techniques.

CHAPTER 2: FINANCIAL ENGINEERING

2.1 Definitions and Concepts

2.1.1 Derivatives

A derivative is a financial instrument whose value depends on an underlying variable.[7] There are many different types of derivatives but the most common are options, futures, and swaps. Although derivatives have no intrinsic value, as they are not stand-alone assets, many are traded as if they are assets.

The two basic types of derivatives are vanilla (simple and more common) and exotic (more complicated and specialized) derivatives. Derivatives are typically categorised by four characteristics:

a. The **market** in which the derivative is traded (exchange-traded or over-the-counter);
b. The **relationship** between the derivative and the underlying asset (swap, option, future, forward, etc.);
c. The **nature** of the underlying asset (foreign exchange derivative, equity derivative, interest rate derivative etc.);
d. The **pay-off** profile of the derivative (call, put, long, short, etc).

2.1.2 Uses of Derivatives

Derivatives are commonly used for the following reasons:

a. **Optionality** – to create an option so that the value of the derivative is linked to a specific condition.
b. **Gearing** – provides leverage (gearing) so that a small movement in the underlying asset can generate a large increase in the value of the derivative.
c. **Hedging** - to mitigate risk in the underlying asset using a derivative which moves in the opposite direction.
d. **Speculation** - to make a profit if the underlying asset moves as expected.
e. **Exposure** – to derive a financial benefit from an underlying asset when it is not possible to trade in the underlying asset.

[7] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pg.1.

2.1.3 Types of Traders

The three most common types of traders are hedgers, speculators, and arbitrageurs.

a. **Hedgers** – use derivative instruments to reduce the risk they potentially face from movement in the underlying asset variable.
b. **Speculators** – use derivative instruments to potentially benefit from movements in the underlying variable.
c. **Arbitrageurs** – use derivative instruments to obtain a riskless benefit by transacting in two or more markets simultaneously.

2.1.4 Exchange-Traded and Over-the-Counter (OTC) Derivatives

In general, derivative contracts can be segmented into two groups depending on the manner in which they are traded in the market.

2.1.4.1 Exchange-Traded Derivatives

Exchange-traded derivative contracts are traded via specialized derivatives exchanges. Financial instruments traded on a derivatives exchange are standardised by the exchange. The exchange acts as an intermediary between the parties and takes an initial margin to act as a guarantee. The world's largest derivatives exchanges are the Korean Exchange (which lists KOSPI Index Futures & Options), Eurex (which lists a wide range of European products such as interest rate & index products), and CME Group (made up of the 2007 merger of the Chicago Mercantile Exchange and the Chicago Board of Trade and the 2008 acquisition of the New York Mercantile Exchange). According to BIS, the combined turnover in the world's derivatives exchanges totalled US$ 344 trillion during the fourth quarter of 2005.

2.1.4.2 Over-the-Counter (OTC) Derivatives

OTC derivative contracts are traded directly between the parties (typically using the telephone or computers) without going through an exchange or intermediary. The advantage of an OTC trade is that the terms of the transaction can be more specific than those specified by an exchange. The OTC derivative market is the largest market for derivatives. According to the Bank of International Settlements (BIS), the total outstanding value of the OTC market as of 2008 was US$684 trillion.[8] The primary risk associated with OTC trades is counter-party risk as there is the possibility that the contract will not be honoured.

[8] The Bank for International Settlements (BIS), *Semi-Annual OTC Derivatives Statistics Report: June 2008.*

2.1.5 Common Derivative Contract Types

There are three major classes of derivatives:

a. **Options** - are contracts that give the owner the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset.

b. **Futures/Forwards** – are contracts to buy or sell an asset on or before some future date at a price that is specified today. The difference between a futures contract and a forward contract is that a futures contract is a standardized contract written by an exchange while a forward is a non-standardized contract written by the parties themselves.

c. **Swaps** - are contracts in which two counter-parties agree to exchange one stream of cash flows for another on or before a specified future date based on some underlying variable (e.g. currencies, exchange rates, bonds, interest rates, commodities, stocks, etc.).

2.2 Options

2.2.1 Definitions and Concepts

a. **Call option** - the right but not the obligation to buy an underlying asset at a certain price and can be used to reduce risk in cases where there is the fear that the future price of the underlying asset will rise.

b. **Put option** - the right but not the obligation to sell an underlying asset at a certain price and can be used to reduce risk in cases where there is the fear that the future price of the underlying asset will fall.

c. **Price of an option** - derived from the difference between the reference price and the value of the underlying asset plus a premium based on the time remaining until the expiration of the option.

d. **Strike price or exercise price** – the reference price at which the underlying asset may be traded.

e. **Exercising** - activating an option and thus trading the underlying asset at the agreed-upon price.

f. **Writing an option** - granting the option.

g. **Premium**- the price of the option.

h. **In-the-money** – either a call (put) option where the asset price is greater (less) than the strike price.

i. **Out-of-the-money** - either a call (put) option where the asset price is less (greater) than the strike price.

j. **Deep-in-the-money** - An option which is so far in the money that it is unlikely to go out of the money prior to expiration.

2.2.2 Types of Options

The naming conventions used in option contracts are used to identify the key properties associated with the various types of options.

a. **European option** - an option that can only be exercised on the expiration date.

b. **American option** - an option that can be exercised on or before the expiration date.

c. **Bermudan option** - an option that may be exercised only on specified dates on or before the expiration date.

d. **Barrier option** - any option where the underlying asset's price must breach a predetermined level or 'barrier' before it can be exercised.

e. **Exotic option** – a non-standard option.

f. **Vanilla option** - an option that is not exotic.

2.2.3 Exotic Options[9]

With the advent of complex financial engineering, various complex options have been designed for hedging purposes; for tax, accounting, legal or regulatory purposes; to reflect the outlook for future movements in the market; or to capture greater potential benefit than offered by vanilla options. The most common exotic options are as follows:

a. **Asian Options** – where the payoff depends on the average price of the underlying asset over a specified period of the option's life.

b. **Barrier Options** – where the payoff depends on whether the price of the underlying asset has breached a predetermined level.

c. **Basket Options** – a popular form of rainbow option where the payoff is dependant on a portfolio (basket) of assets.

d. **Bermudan Options** – an option that can only be exercised on specific dates during its life.

e. **Binary Options** – an option with a discontinuous payoff.

f. **Chooser Options** – allows the holder of the option after a period of time to choose whether the option is a call or a put.

g. **Compound Options** – an option on options. There are four types of compound options: a call on a call, a call on a put, a put on a put, and a put on a call.

h. **Exchange Options** – an option to exchange one asset for another.

i. **Forward Start Options** – an option that will only commence at a specified date in the future.

j. **Lookback Options** – where the payoff is dependant on the maximum or minimum of the asset price achieved during the life of the option.

[9] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 19.

k. **Packages** – a portfolio that consists of standard call and put options, possibly combined with forward contracts, cash, and the underlying asset itself.

l. **Rainbow Options** – an option with a payoff that is dependant on two or more underlying variables.

m. **Shout Options** – where the option holder has the right (can *shout*) to lock in the minimum value for the payoff at one time during the life of the option. At expiration, the option holder then receives either the payoff from a European option or the intrinsic value at the time of the shout (whichever is higher).

2.2.4 Exchange-Traded and OTC Options

The most common exchange-traded options include:

- stock options
- commodity options
- bond options
- interest rate options
- index options
- options on futures contracts
- callable bull/bear contract

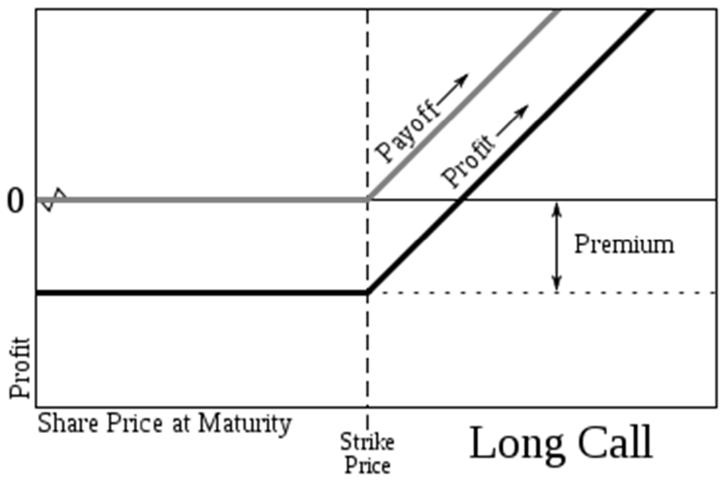
Option types commonly traded over the counter include:

- interest rate options
- currency cross rate options
- options on swaps (swaptions)

2.2.5 The Basic Option Trades

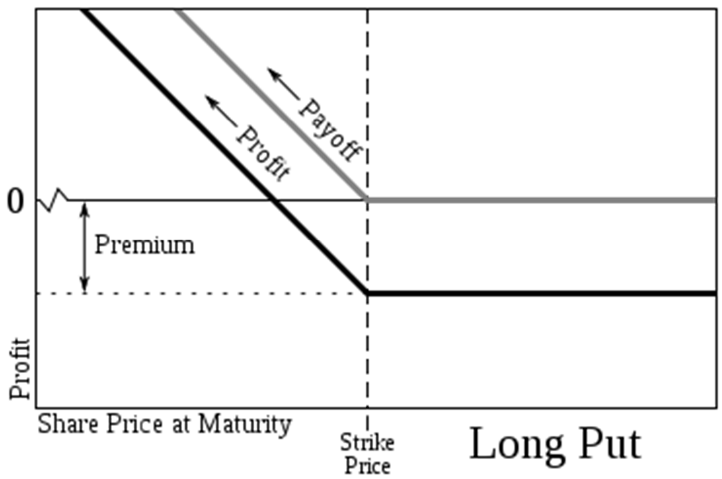
Every option trade has two sides. On the one side there is the buyer of the option (long position) and on the other side there is the seller of the option (short position). Thus there are four basic option positions:

2.2.5.1 Long Call



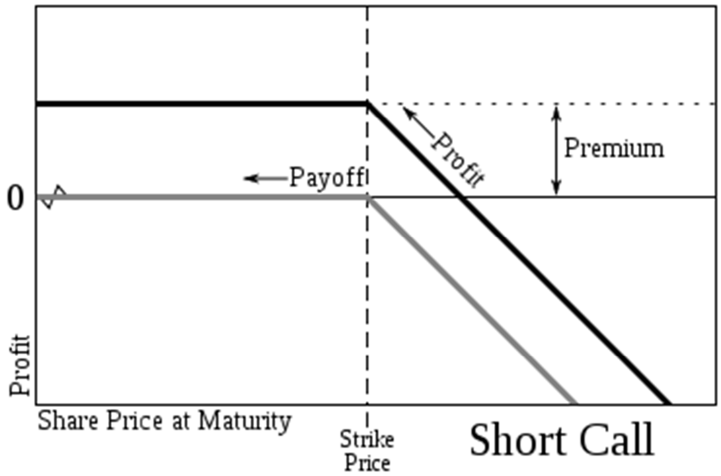
A long call is used when a trader believes that the share price will increase so he buys the right to purchase the share (a call option). Then, if the share price at expiration is higher than the exercise price plus the premium paid for the option, the trader will make a profit. In contrast, if the share price at expiration is lower than the exercise price, the trader just lets the call option expire and only loses the premium he paid to obtain the option.

2.2.5.2 Long Put



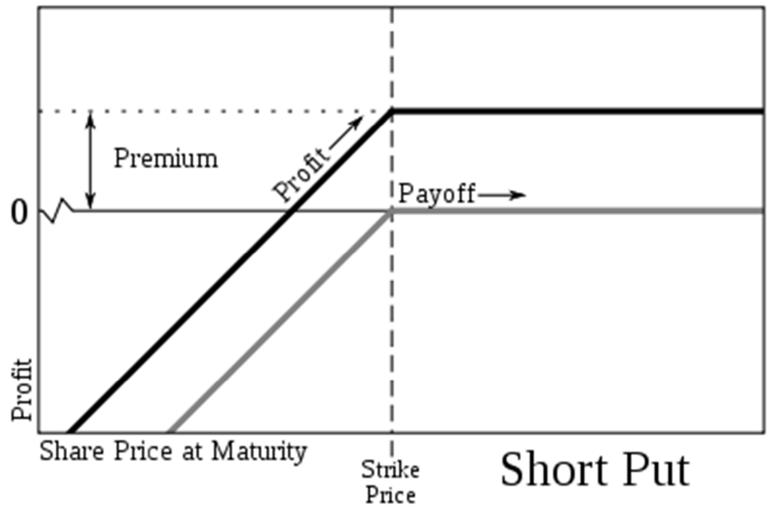
A long put is the opposite of a long call and thus is used if a trader believes that the share price will decrease so he buys the right to sell the share at a fixed price (a put option). Then, if the share price at expiration is below the exercise price plus the premium paid for the option, the trader will make a profit. In contrast, if the share price at expiration is above the exercise price, the trader just lets the put option expire and only loses the premium he paid to obtain the option.

2.2.5.3 Short Call



A short call is used when a trader believes that the share price will decrease so he sells the share short by *writing* a call option (selling short is where the trader sells shares that he does not own). In contrast to the long position, a trader selling a short call has an obligation to sell the asset to the call buyer at the buyer's option. If the share price decreases as expected, the short call position will make a profit by the amount of the premium. However, if the share price increases above the exercise price in excess of the premium paid, then the short will lose money with the potential loss increasing as the share price decreases.

2.2.5.4 Short Put



A short put is the opposite of a short call and thus is used when a trader believes that the price of a share will increase so he buys the share or instead sells (*write*) a put. As in the case of the short call option, the trader selling a short put has an obligation to buy the asset from the put buyer at the put buyer's option. If the share price at expiration is above the exercise price then the short put position will make a profit by the amount of the premium. However, if the share price at expiration is below the exercise price by more than the premium then the trader will lose money, with the potential loss being the full value of the share.

2.2.6 Hedging with Options[10]

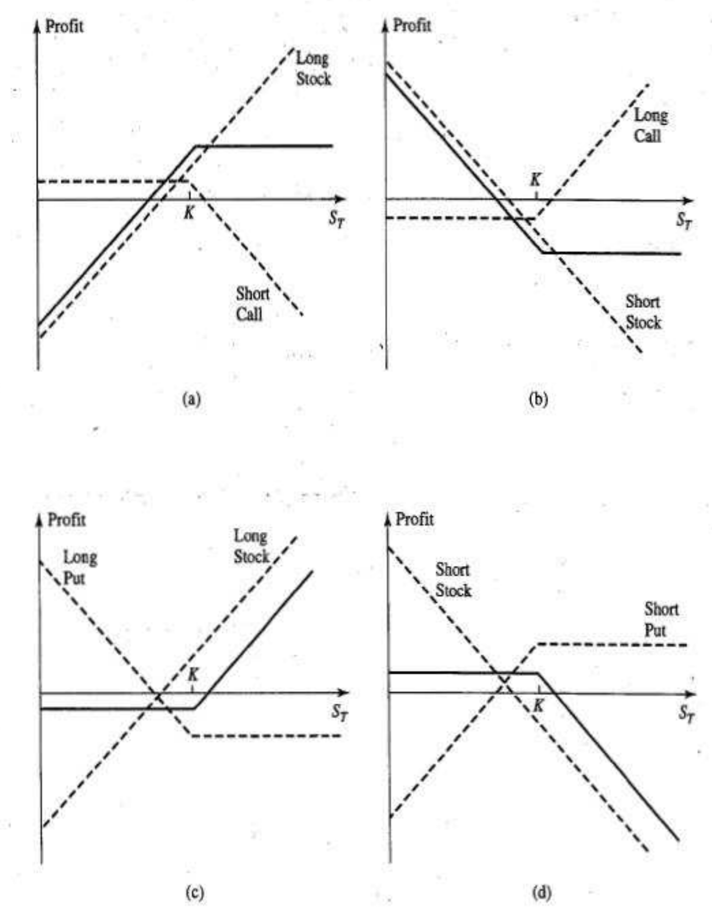
A variety of option hedges can be formulated using a combination of the basic option trades.

2.2.6.1 Hedges Involving a Share and an Option

Options can be combined with shares to produce four hedging strategies:

a. A long position in a share can be combined with a short position in a call (called *writing a covered call*).
b. A short position in a share can be combined with a long position in a call.
c. A long position in a put can be combined with a long position in the share (called *a protective put*).
d. A short position in a put can be combined with a short position in a share.

[10] Hull, J.C. (2003), *Options, Futures, and Other Derivatives*, 5th ed., Prentice Hall, Chapter 9. Additional figures obtained from theoptionsguide.com

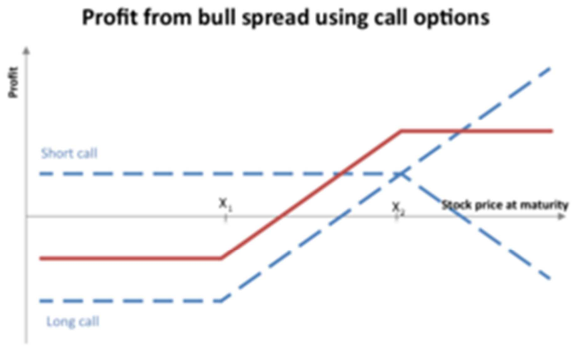


Source: Hull, J.C. (2003), *Options, Futures, and Other Derivatives*, Fifth Edition, Prentice Hall, Pg. 186.

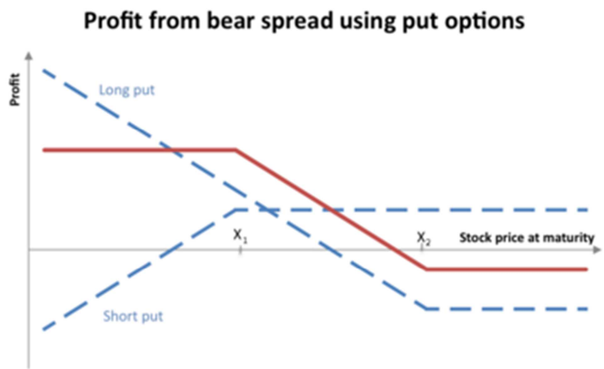
2.2.6.2 Spreads

Spreads consist of taking a position in two or more options of the same type:

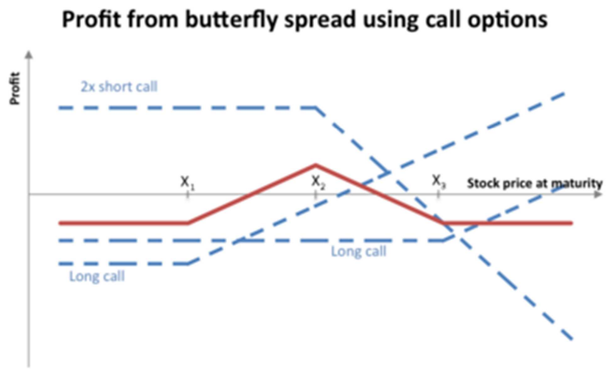
a. **Bull spreads** – used when a trader hopes that the share price will rise but wants to limit upside and downside risk. Bull spreads are created by buying a call option on a share and selling a call option on the same share with a higher strike price. A bull spread can also be created by buying a put option with a low strike price and selling a put with a high strike price.



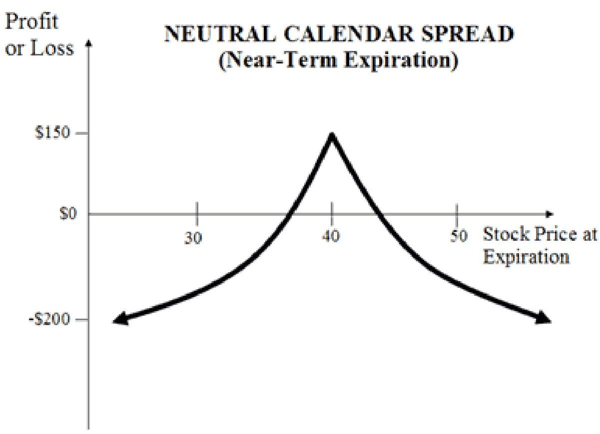
b. **Bear spreads** – used when a trader hopes that the share price will fall but wants to limit upside and downside risk. Bear spreads are created by buying a call option with one strike price and selling a call option with a strike price that is lower than the first call option. Bear spreads can also be created using put options by buying a put option with a high strike price and selling another put option with a lower strike price.



c. **Butterfly spreads** – used when a trader believes that large share price movements are unlikely. Butterfly spreads make use of options with three different strike prices. Butterfly spreads are created by buying a call option with a low strike price (S1), buying a call option with a high strike price (S3), and selling two call options with strike prices that are halfway between S1 and S3 and close to the current share price (S2). Butterfly spreads lead to a profit when the share price remains close to S2 and leads to a small loss if the share price moves in either direction.



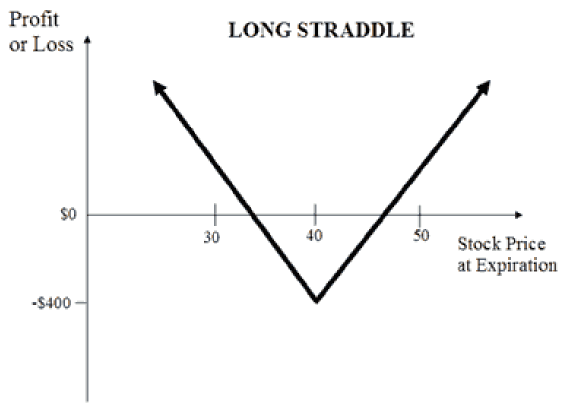
d. **Calendar spreads** – consist of options with the same strike price but different expiration dates. Calendar spreads are created by selling a call option with a certain strike price and buying a second call option with a longer maturity and the same strike price (the longer the maturity the more expensive the option). Calendar spreads can also be created using put options by buying a long-maturity put option and selling a short-maturity put option. The trader of a calendar spread will make a profit if the share price at expiration of the shorter maturity option is close to the strike price of the short-maturity option. However, the trade will be at a loss if the share price is significantly above or below the strike price. The calendar spread is *neutral* if the strike price is close to the current share price, *bullish* if the strike price is higher than the current share price, *bearish* if the strike price is lower than the current strike price and *reverse* if the trader buys a short-maturity option and sells a long-maturity option.

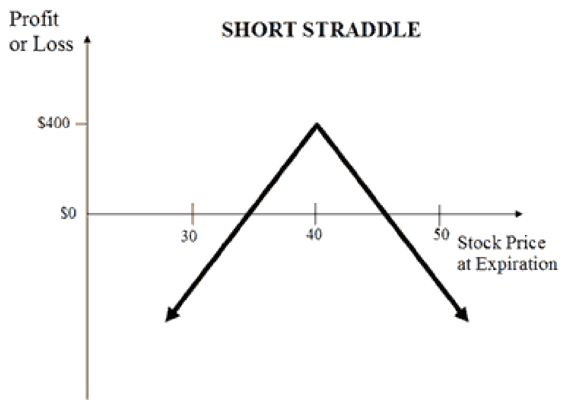


2.2.6.3 Combinations

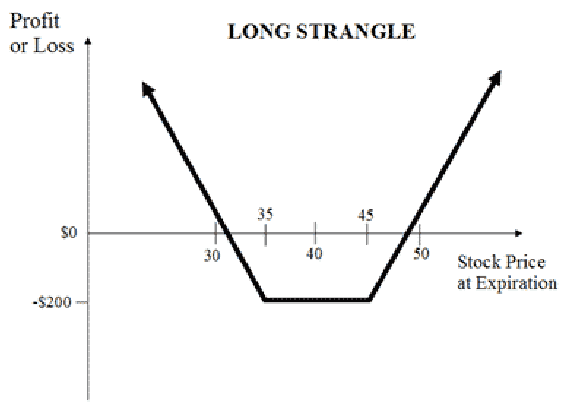
A 'combination' refers to a strategy involving calls and puts on the same share.

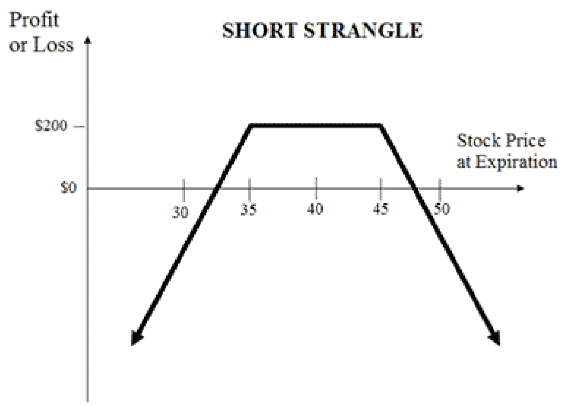
a. **Straddle** – used when the trader believes that there will be a significant movement in the share price but does not know in which direction. A straddle consists of buying a call and a put option with the same strike price and expiration date. If the share price at expiration is close to the strike price then the straddle will lead to a loss but if the share price moves significantly in either direction then the straddle will produce a substantial profit.



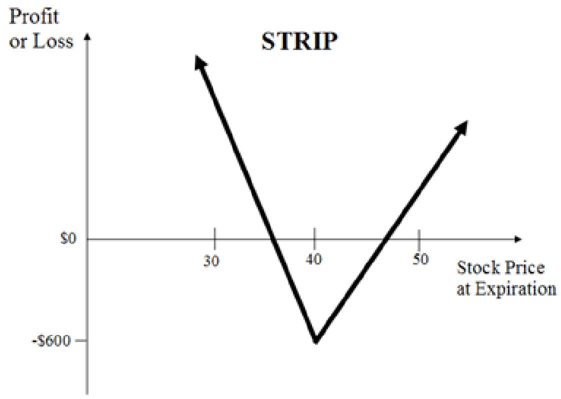


b. **Strangles** – consists of buying a put and a call option with the same expiration date but different strike prices. A strangle is similar to a straddle in that the trader believes that there will be a significant movement in the share price but does not know in which direction. However, in a strangle the share price has to move further than in a straddle and thus there is less downside risk.

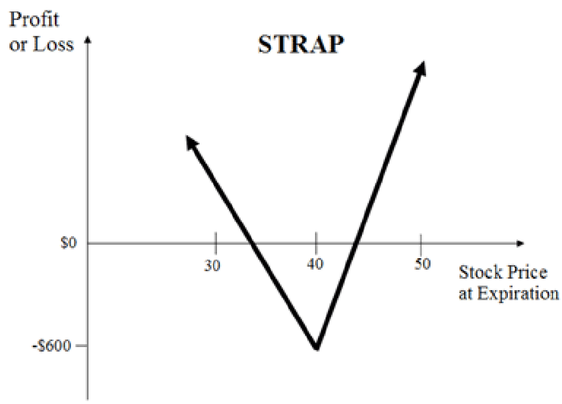




c. **Strips –** used when the trader believes that there will be a large downward movement in the share price. A strip consists of a long position in one call and two puts with the same strike price and expiration dates.



d. **Straps** – used when a trader believes that there will be a large increase in the share price. Straps are the opposite of strips and consist of a long position in two calls and one put with the same strike price and expiration dates.



2.2.7 The Greeks[11]

A financial institution that sells an over-the-counter option will possibly have difficulty managing the risks, particularly in the case of exotic options. Thus the 'Greeks' have been developed to manage the risks associated with particular aspects of holding an OTC option.

a. **Delta** (Δ) - measures the extent to which the option's value varies with changes in the underlying price. Delta is measured by the ratio $\Delta = \dfrac{\textit{Change in option's value}}{\textit{Change in underlying's value}}$.

b. **Delta hedging an option** - achieved by buying or selling an amount of the underlying asset so that any detrimental changes in the option are offset by changes in the underlying asset.

[11] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 14.

c. **Gamma** (Γ) - measures the extent to which the delta of an option changes with changes in the underlying asset. Gamma is measured by the ratio $\Gamma = \frac{Change\ in\ \Delta}{Change\ in\ price}$

d. **Gamma-neutral** – to be fully hedged such that a portfolio of options has a delta that does not change.

e. **Vega** – measures the extent to which an option's value changes with changes in the volatility of the underlying asset. Vega is measured by the ratio $Vega = \frac{Change\ in\ option's\ value}{Change\ in\ volatility}$

f. **Theta** (Θ) - measures the extent to which an option's value changes with time to maturity. Theta is measured by the ratio $\Theta = -\frac{Change\ in\ option's\ value}{Change\ in\ time}$

g. **Rho** (ρ) - measures the extent to which an option's value changes with changes in interest rates. Rho is measured by the ratio $\rho = \frac{Change\ in\ option's\ value}{Change\ in\ interest\ rates}$

2.2.8 Valuation Models

A variety of empirical methods are used to value options, which include the Black-Scholes model, stochastic volatility models, binomial tree models, Monte Carlo simulations, finite difference models amongst others.[12]

2.2.8.1 Black-Scholes[13]

Fisher Black, Myron Scholes, and Robert Merton developed the differential equation for modelling European options in the early 1970's.[14] The model assumes that it is possible to design a risk neutral portfolio that replicates the returns of holding an option. At the same time, the model generates the parameters necessary for hedging the risk associated with holding the option. The ideas behind the Black-Scholes model were ground-breaking, leading to the development of financial engineering, hedging strategies, and option pricing. Scholes and Merton eventually received the 1996 Nobel Prize for economics (sadly, Fisher Black had passed away in 1995).

The Black-Scholes formulas for pricing a European option on a non-dividend paying share are as follows:

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[12] Reilly, F.K. and Brown, K.C. (2003), *Investment Analysis and Portfolio Management*, 7th ed., Thomson Southwestern, Chapter 23.

[13] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 12.

[14] Black, F. and Scholes, M.S. (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, Vol. 81, No. 3, 637-654; Merton, R.C. "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science,* Vol. 4, Pgs. 141-183.

Call option $= S_0 N(d_1) - Ke^{-rT} N(d_2)$

Put option $= Ke^{-rT} N(-d_2) - S_0 N(-d_1)$

where:

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

S_0	is the current market price of the underlying
N(x)	is the cumulative normal distribution function
K	is the strike price of the option
T	is the time to expiration in years
σ	is the volatility of the underlying asset
r	is the risk-free interest rate

However, despite the mathematical elegance of the Black-Scholes model, the practical application of the model is limited by a set of assumptions:

- Future relative price changes are independent of past changes and the current price;
- Volatility and interest rates are constant;
- The probability distribution of relative price changes is lognormal;
- There are no transaction costs;
- The option can only be exercised at maturity; and
- The underlying asset does not pay out dividends.

Hence, the binomial option model is commonly used to overcome these limitations.

Worked Example:[15]

A three-month call option is excisable at R2.55. The share is currently trading at R2.50, the risk-free rate is 10% and the standard deviation of annual returns on the share is 0.3. What is the value of the call option?

$$d_1 = \frac{\ln\left(\frac{250}{255}\right) + \left(0.1 + \frac{0.3^2}{2}\right)0.25}{0.3\sqrt{0.25}} = 0.10925$$

$$d_2 = \frac{\ln\left(\frac{250}{255}\right) + \left(0.1 - \frac{0.3^2}{2}\right)0.25}{0.3\sqrt{0.25}} = d_1 - 0.3\sqrt{0.25} = -0.04035$$

From the tables for the standard normal distribution function:

$N(d_1) = 0.50 + 0.0438 = 0.5438$ and $N(d_2) = 0.50 + (-0.0160) = 0.4840$

Thus the value of the call option is:

$\text{Call option} = 250N(d_1) - 255e^{-0.1\times 0.25}N(d_2) = 15.6\,\text{cents}$

2.2.8.2 Binomial Tree Pricing Model

The standard binomial lattice model was first proposed by John Cox, Stephen Ross and Mark Rubinstein in 1979 and is a discrete simulation process that plots the option's price movements over time.[16] The model uses a binomial tree of discrete future possible underlying stock prices to construct a riskless portfolio consisting of an option and stock (as in the Black-Scholes model). Using a simple formula, the option price at each node in the tree can then be determined. This value approximates the theoretical value produced by the Black-Scholes model but is more accurate as it allows for the introduction of early exercise (American options), non-exercisable periods (vesting), dividends, and changing inputs (such as varying volatility and interest rates over the life-time of the option).

[15] Example sourced from Correia, C., Flynn, D., Uliana, E., Wormald, M. (2010), *Financial Management*, 7th ed., Juta & Company Ltd., South Africa, Chapter 18, Pgs. 12-13.

[16] Cox, J.C., Ross, A.S., and Rubinstein, M. (1979), "Option Pricing: A Simplified Approach," *Journal of Financial Economics,* No. 7, pgs. 229-263.

The first step when using a binomial lattice is to model the value of the underlying asset (*S*), which entails inputting the *up* and *down* bifurcations into the binomial lattice:[17]

$u = e^{\sigma\sqrt{\Delta T}}$ (*up* bifurcation) $d = \frac{1}{u}$ (*down* bifurcation)

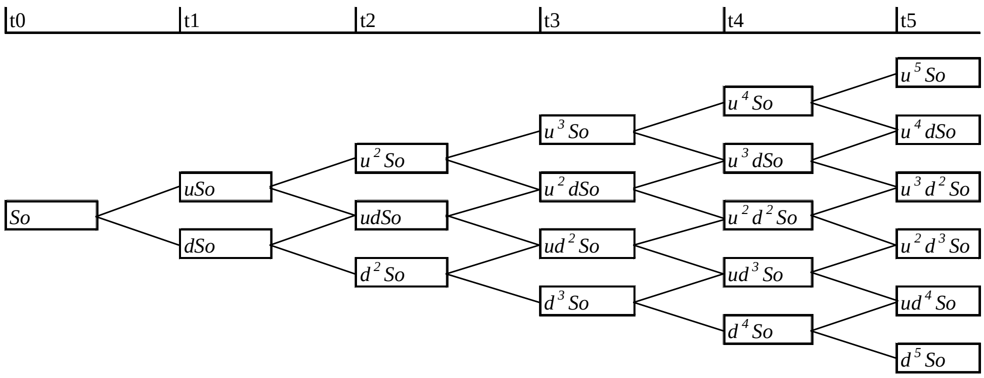
where:

$\Delta t = \frac{T}{n}$

n is the number of time steps

σ is the volatility

Multiplicative binomial lattice:



Once the underlying asset and the terminal values have been determined, the next step is to value the option itself by applying a backward induction process through the lattice, starting with the terminal nodes and working towards *So* using the following equations:

$$S_0 = [pu + (1-p)d]e^{-r(\Delta T)}$$

The asset price at each node is then set equal to $Su^i d^{j-i}$ where *i=0,1,…,j* and thus the put and call options are formulated by the following:

Put option $P_{j,i} = \max\left\{X - Su^i d^{j-i}, e^{-r\Delta t}[p(P_{j+1,i+1} + (1-p)P_{j+1,i})]\right\}$

Call option $P_{j,i} = \max\left\{Su^i d^{j-i} - X, e^{-r\Delta t}[p(P_{j+1,i+1} + (1-p)P_{j+1,i})]\right\}$

[17] Mun, J. (2006), *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions,* 2nd ed., John Wiley & Sons, New Jersey, Pg. 128.

where:

p is the risk-neutral probability measured as $p = \frac{e^{b\Delta t} - d}{u - d}$

r is the risk-free rate

b is the cost of carry

Worked Example: [18]

An American stock put option has six months to expiry. The stock price is R100, the strike price is R95, the risk-free interest rate is 8%, and the volatility is 30%. Calculate the value of this option using a binomial tree with five time steps.

Thus $\Delta t = \dfrac{0.5}{5} = 0.1$

$u = e^{0.3\sqrt{0.08}} = 1.0995$ and $d = e^{-0.30\sqrt{0.08}} = 0.9095$

Hence the probability of the stock price increasing at the next time step is given by:

$$p = \frac{e^{0.08 \times 0.1} - 0.9095}{1.0995 - 0.9095} = 0.5861$$

and thus the 5-step binomial tree finds that the value of the option is R4.92

					160.70
				146.15	0.00
			132.92	0.00	132.92
		120.89	0.00	120.89	0.00
	109.95	0.44	109.95	0.00	109.95
100.00	2.02	100.00	0.92	100.00	0.00
4.92	90.95	3.75	90.95	1.93	90.95
	8.13	82.72	6.86	82.72	4.05
		12.97	75.23	12.28	75.23
			19.77	68.42	19.77
				26.58	62.23
					32.77

2.2.8.3 Other Option Valuation Techniques

Since the 1987 stock market crash, it has become apparent that the volatility used to model options tends to be stochastic, varying over time and for the price of the underlying asset. This effect gives rise to the *volatility smile* of options. Hence, stochastic volatility models have been developed to take account of the two stochastic variables of the share price and its volatility. The

[18] Example sourced from Espen Gaarder Haug, E. G. (1998), *The Complete Guide to Options Pricing Formulas*, McGraw-Hill Companies, USA., Chapter 3.1.1, Pgs. 112-113.

most common approaches are the *implied volatility function* (*IVF*) model,[19] the *implied tree model*[20] and the Heston model.[21] Unfortunately, stochastic volatility models can be computationally complex.

In many cases, the valuation of options using standard option pricing techniques may not be possible. Hence in these cases, a Monte Carlo simulation may be more applicable. Instead of solving a series of complex differential equations, a Monte Carlo model simulates the random price paths of the underlying asset, resulting in a payoff for the option. The average of these payoffs can then be discounted to produce the expected value of the option yield. However, the limitations associated with using Monte Carlo simulation to value options are that the computation time necessary to achieve sufficiently accurate results may be too high, and the approach can not easily value American path-dependant options.[22]

2.3 Futures

2.3.1 Definitions and Concepts

A futures contract is a contract to buy or sell an asset at a price decided when the contract is entered into. However, unlike an option, a futures contract entails the right and obligation to trade for both parties. Futures contracts are exchange-traded derivatives where the exchange's clearing house acts as the counter-party on all contracts, sets margin requirements, and provides the mechanism for settlement. The party agreeing to buy the underlying asset assumes a long position while the party agreeing to sell the asset assumes a short position. Futures contracts are *market* to *market* daily (priced). The *settlement price* is the official price of the futures contract at the end of a day's trading session on the exchange. The future date agreed to by both parties is called the *delivery date* or *final settlement date*.

Contracts on financial instruments were introduced in the 1970s by the Chicago Mercantile Exchange (CME). Since then 90 futures and options exchanges have been established world-wide,[23] including:

- CME Group (formerly CBOT and CME)
- Deutsche Terminbörse (now called Eurex)

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[19] Derman,E. and Kani, I. (1994), "Riding on a Smile," *RISK*, Pgs. 32-39; Anderson, L.B.G. and Brotherton-Ratcliffe, R. (1997), "The Equity Option Volatility Smile: An Implicit Finite Difference Approach," *Journal of Computational Finance*, Vol. 1, No. 2, Pgs. 5-37.

[20] Dupire, B. (1994), "Pricing with a Smile," *RISK*, Pgs. 18-20; Rubinstein, M. (1994), "Implied Binomial Trees," *Journal of Finance*, Vol. 49, No. 3, Pgs. 771-818.

[21] Heston, S.L. (1993), "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," *The Review of Financial Studies*, Vol. 6, Pgs. 327–343.

[22] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pg.462.

[23] *Futures & Options Factbook*. Institute for Financial Markets, http://www.theIFM.org/gfb

- Dubai Mercantile Exchange
- Intercontinental Exchange (ICE Futures Europe)
- Korean Exchange
- London International Financial Futures Exchange (now called Euronext.liffe)
- London Metal Exchange
- New York Mercantile Exchange
- NYSE Euronext (which absorbed Euronext)
- Osaka Securities Exchange
- Singapore Exchange
- South African Futures Exchange
- Sydney Futures Exchange
- Tokyo Commodity Exchange
- Tokyo Financial Exchange
- Tokyo Stock Exchange

2.3.2 Components

Futures contracts typically consist of eight specifications:

a. The nature of the underlying asset;
b. The type of settlement (cash or physical);
c. The amount and units of the underlying asset per contract;
d. The currency in which the futures contract is quoted;
e. The quality and quantity to be delivered;
f. The month of delivery;
g. The last trading date; and
h. Other contract-specific details.

2.3.3 Margins

In order to minimise counter-party risk, all trades undertaken on a regulated futures exchange are guaranteed by a clearing house. The clearing house acts as a buyer for each seller, and as a seller for each buyer and thus in the event of a counterparty default, the clearing house absorbs the risk of a loss. So as to minimise the clearing house's credit risk exposure, futures traders are required to post a margin (which is commonly 5%-15% of the value of the futures contract).

The primary forms of margins are as follows:

a. **Clearing margin** – a margin posted by a member of the clearing house to ensure that companies deliver on their customers' open futures and options contracts.
b. **Customer margin** – margins required from buyers and sellers of futures contracts to ensure fulfilment of the futures contract obligations. Customer margin accounts are overseen by Futures Commission Merchants.
c. **Initial margin** – the payment required from a futures trader at the time of the trade. Initial margin is set by the exchange.
d. **Margin call** – if there is a loss arising from the futures contract or when the initial margin is being eroded, a broker can order the trader to restore the amount of initial margin available. Margin calls are usually expected to be paid and received on the same day. If not, the broker has the right to close sufficient positions to make up the amount required and the trader is liable for any remaining deficit.
e. **Maintenance margin** – a term used by U.S. exchanges to define by how much the value of the initial margin can reduce before a margin call is made. If the margin drops below the margin maintenance requirement then a margin call will be made to bring the account back up to the required level.
f. **Margin-equity ratio** - a term used by speculators to measure the amount of trading capital held as margin at any particular time.

2.3.4 Hedging with Futures[24]

Futures contracts are often used to hedge risk exposure by removing the uncertainty about the future price of an asset. This involves buying a futures contract so as to lock in the price of an asset that will be bought or sold in the future and thus eliminate the ambiguity of future profits or losses.

Hence the two common hedging strategies used are short hedges and long hedges.

a. A **short hedge** is applicable when a company knows that it will be selling an asset at some future date. The company can then take a short position in a futures contract to hedge its position. For example, if a company must fulfil a contract to sell 20,000 bushels of maize in six months time and the spot price is $2.50 per bushel and the futures price is $2.35 per bushel, the company will short a futures contract on maize and close out the futures position in six months. In this case, the company has reduced its risk by ensuring that it will receive $2.35 for each bushel of maize that it sells.
b. A **long hedge** is applicable when a company knows that it will be purchasing an asset in the future. The company can then take a long position in a futures contract to hedge its position.

[24] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 4.

For example, if an airline knows that in six months it will have to buy 1 million gallons of aviation fuel and the spot price for avgas is $6.25 per gallon and the six-month futures price is $6.15 per gallon, by buying the futures contract, the airline can lock in a price of $6.15 per gallon thus reducing the airline's risk because it will be able close its futures position and buy 1 million gallons of avgas for $6.15 per gallon in six months.

2.3.5 Futures versus Forwards[25]

Futures and forwards are both contracts to deliver an asset on a future date at a pre-arranged price. However, there are fundamental differences between the two:

- Futures are exchange-traded while forwards are traded over-the-counter;
- Futures are standardized while forwards are customized;
- Futures have a range of delivery dates while forwards usually have one delivery date;
- Futures are margined while forwards are not;
- Futures are settled daily while forwards are settled at the end of the contract;
- Futures contracts are usually closed out prior to maturity while forwards require delivery or cash-settlement;
- Forwards are more risky than futures because unlike futures, forwards are not margined daily. Hence, due to movements in the price of the underlying asset, a large differential can build up between the forward's delivery price and the settlement price;
- Forwards have credit risk because the supplier may be unable to deliver the asset or the buyer may be unable to pay for it on the delivery date or the closing date. In the case of an exchange-traded future, the clearing house interposes itself on every trade and thus there is no risk of counterparty default.

2.3.6 Valuation of Futures[26]

2.3.6.1 The Generalised Equation

The general equation for valuing a future contract with no income (*F*) is:

$$F_0 = S_0 e^{rT}$$

where:

S is the price of the underlying asset

[25] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pgs. 36-37.
[26] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 3.

r is the risk-free interest rate

T is the time to maturity

Worked Example:[27]

Calculate the value of the futures contract on a non-dividend paying stock where the maturity date is in three months, the current asset price is R100, and the three-month risk-free rate is 7% per annum.

Thus the futures price is R101.76 calculated as:

$$F = 100e^{0.07 \times 0.25} = 101.76$$

2.3.6.2 Known Income

The equation for calculating the futures contract when an investment asset will provide income with a present value I during the life of the contract is:

$$F_0 = (S_0 - I)e^{rT}$$

Worked Example:[28]

Calculate the value of a one-year forward contract on a two-year bond where the two-year bond's price is R800, the delivery price is R820, and two coupons of R50 will be paid in 6 and 12 months, respectively. The risk-less interest rate is 8% per annum for 6 months and 9% per annum for 12 months.

First, in order to apply the formula, the value of I must be discounted to the present at an appropriate interest rate and thus in this case the value of I is calculated as R93.73 as follows:

$$I = 50e^{-0.08 \times 0.5} + 50e^{-0.09 \times 1} = 48.039 + 45.696 = 93.73$$

and thus the forward price is determined to be R772.78 by applying the future with known income equation:

$$F = (800 - 93.735)e^{0.09 \times 1} = 772.78$$

2.3.6.3 Known Yield

In a case where the investment asset provides an average annual yield q rather than a cash income during the life of the contract then the equation becomes:

[27] Example sourced from Bellalah, M. (2010), Derivatives, *Risk Management & Value*, World Scientific Publishing Co., Singapore, Chapter 2.2.3., Pg. 74.
[28] Example sourced from ibid, Chapter 2.2.3.6, Pg. 75.

$$F_0 = S_0 e^{(r-q)T}$$

Worked Example: [29]
Calculate the value of a three-month futures contract on a security that provides a continuous dividend yield of 5% per annum. The current asset price is R100 and the risk-free rate is 7% per annum.

Thus the futures price is calculated to be R100.50 by applying the future with known yield equation:

$$F = 100e^{(0.07-0.05)0.25} = 100.50$$

2.3.6.4 Currency Futures

When valuing a futures foreign currency contract the equation is:

$$F_0 = S_0 e^{(R-r)T}$$

Where R is the domestic risk-free interest rate and r is the foreign risk-free rate. Note that when the foreign interest rate is greater than the domestic interest rate ($r > R$), F is less than S and F decreases as the time to maturity (T) increases. In the case where the domestic interest rate is greater than the foreign interest rate ($r < R$), the opposite will apply. The cost of carrying a foreign currency is given by the difference between the domestic riskless rate (R) and the foreign riskless rate (r) as measured by R-r.

Worked Example: [30]
Calculate the value of a three-month futures contract on a foreign currency where the spot price is R180, the domestic risk-free rate is 7% per annum, and the foreign risk-free rate is 6% per annum.

Thus the futures price is calculated as R180.45 from the following equation:

$$F = 180e^{(0.07-0.06)0.25} = 180.45$$

2.3.6.5 Commodity Futures

When valuing commodities futures, the generalised futures equation must be adjusted for the cost or benefit of storing or holding the asset. In the case where there is a cost associated with storing the asset then the equation is:

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[29] Example sourced from ibid, Chapter 2.2.3.4 Pg. 74.
[30] Example sourced from ibid, Chapter 2.2.3.5 Pg. 75.

$$F_0 = (S_0 + U)e^{rT}$$

Where U is the present value of all the storage costs incurred during the life of the futures contract:

$$U = Ce^{-rT}$$

Worked Example: [31]

Calculate the value of a one-year futures contract on gold where the cost of carry is R3 per ounce paid at the end of the year, the spot price is R500, and the risk free rate is 10% per annum.

First, one must calculate the value of U, which is found to be R2.71 as follows:

$$U = 3e^{-0.1(1)} = 2.71$$

Thereafter, the futures price is calculated as R555.58 using the equation for a commodity future:

$$F = (500 + 2.7145)e^{0.1(1)} = 555.58$$

However, if there is a benefit from holding the asset y ('convenience yield'), as well as a storage cost then the equation is:

$$F_0 = S_0 e^{(r+u-y)(T-t)}$$

where:

F	is the futures price
S	is the current spot/cash price of the underlying commodity
r	is the risk-free rate of return
u	is the storage cost per unit time as a percent of the asset value
y	the convenience yield
T-t	the time to maturity of the contract

31 Example sourced from ibid, Chapter 2.2.3.5 pg. 76

Worked Example:[32]
Calculate the value of a six month futures contract on gold where the current price of an ounce of gold is R800, the risk-free rate is 6%, the cost of storage is 2% of the purchase price, and the lease rate to lend gold is 1%.

$$F = R800e^{(0.06+0.02-0.01)(0.5)} = R828.50$$

2.4 Swaps

2.4.1 Definitions and Concepts

A swap is defined as a derivative in which two counterparties agree to exchange some form of financial benefit in the future. The swap agreement defines the method of calculation and the dates when the cash flows are to be paid . Usually, when the contract is initiated, at least one of the cash flows is determined by an uncertain variable (such as an interest rate, a foreign exchange rate, an equity price, etc). Swaps are commonly used to hedge risks or to speculate on changes in the expected direction of underlying prices.[33]

2.4.2 Hedging with Swaps

The five types of swaps are:

a. **Interest rate swaps** – an exchange of a fixed interest rate on a notional principle for a floating interest rate on the same notional principle.
b. **Currency swaps** – where principle and interest in one currency are exchanged for principle and interest in another foreign exchange.
c. **Credit default swaps** – gives the holder the right to sell a bond for its face value when the issuer defaults.
d. **Commodity swaps** – an agreement to exchange a fixed quantity of a commodity at a fixed price and a time in the future.
e. **Equity swaps** – where the return on a portfolio is exchanged for a fixed or floating interest rate.

2.4.2.1 Interest Rate Swaps

The most common type of swap is a 'plain vanilla' interest rate swap.[34] In an interest rate swap, a fixed interest rate can be swapped for a floating interest rate. The floating exchange rate commonly used in many interest rate swap transactions is the London Interbank Offer Rate (LIBOR).

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32 Ibid.
33 Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pg. 125.
34 Ibid, Pg 125.

Companies will typically engage in a swap transaction to benefit from comparative advantage. Some companies may have a comparative advantage in fixed interest rate markets while other companies may have a comparative advantage in floating interest rate markets. Thus when a company wants to lower the cost of borrowing, it will go to the market where it has a comparative advantage. However, in many cases the comparative advantage may be in a fixed interest rate market when the company wants a floating interest rate or visa versa and thus the company will undertake a swap to overcome this limitation. Normally the companies engaged in the interest rate swap exchange do not swap payments directly to each other. Instead, each company arranges a separate swap with a financial intermediary. The financial intermediary is often a bank, which benefits financially from the swap by taking a spread from the swap payments.

The value of an interest rate swap where floating (B_{fl}) is received and fixed (B_{fix}) is paid is:

$$V_{swap} = B_{fl} - B_{fix}$$

The value of an interest rate swap where fixed is received and floating is paid is:

$$V_{swap} = B_{fix} - B_{fl}$$

The value of the fixed and floating bonds can then be determined from the following equations:[35]

$$B_{fix} = \sum_{i=1}^{n} k e^{-r_i t_i} + L e^{-r_n t_n}$$

$$B_{fl} = (L + k^*) e^{-r_1 t_1}$$

Where:

B_{fl} is the value of a floating interest rate bond underlying the swap

B_{fix} is the value of a fixed interest rate bond underling the swap

t_i is the time until the ith $(1 \leq i \leq n)$ payment is exchanged

L is the notional principle in the swap agreement

r_i is the LIBOR zero rate corresponding to maturity

[35] Ibid, Pg. 137.

K is the fixed payment made at each payment date

k^* is the floating payment rate

Worked Example:[36]

A municipal issuer and counterparty agree to a $100 million "plain vanilla" swap starting in January 2006 that calls for a 3-year maturity with the municipal issuer paying the Swap Rate (fixed rate) to the counterparty and the counter-party paying 6-month LIBOR (floating rate) to the issuer. Using the above formula, the Swap Rate can be calculated by using the 6-month LIBOR "futures" rate to estimate the present value of the floating component payments. Payments are assumed to be made on a semi-annual basis (i.e., 180-day periods).

The first step is to calculate the present value (PV) of the floating-rate payments:

Time Period	Period Number	Days in Period	Annual Forward Rate	Semi-annual Forward Period Rate	Actual Floating Rate Payment at End Period	Floating Rate Forward Discount Factor	PV of Floating Rate Payment at End of Period
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1/06-6/06	1	180	4.00%	2.000%	$2,000,000	0.9804	$1,960,800
7/06-12/06	2	180	4.25%	2.125%	$2,125,000	0.9600	$2,040,000
1/07-6/07	3	180	4.50%	2.250%	$2,250,000	0.9389	$2,112,525
7/07-12/07	4	180	4.75%	2.375%	$2,375,000	0.9171	$2,178,113
1/08-6/08	5	180	5.00%	2.500%	$2,500,000	0.8947	$2,236,750
7/08-12/08	6	180	5.25%	2.625%	$2,625,000	0.8718	$2,288,475
						PV of Floating Rate Payments=	$12,816,663

Column Description

A= Period the interest rate is in effect
B= Period number (t)
C= Number of days in the period (semi-annual=180 days)
D= Annual interest rate for the future period from financial publications
E= Semi-annual rate for the future period (D/2)
F= Actual forecasted payment (E × $100,000,000)
G= Discount factor=1/[(forward rate for period 1)(forward rate for period 2)...(forward rate for period t)]
H= PV of floating rate payments (F × G)

As with the floating-rate payments, LIBOR forward rates are used to discount the notional principle for the three year period. Hence, in the second step, the present value of the notional principle is calculated by multiplying the notional principle by the days in the period and the floating-rate forward discount factor as follows:

[36] Example sourced from "Understanding Interest Rate Swap Pricing and Math" (January 2007), *CDIAC #06-11*, California Debt and Investment Advisory Commission.

Time Period	Period Number	Days in Period	Annual Forward Rate	Semi-annual Forward Period Rate	Notional Principal	Floating Rate Forward Discount Factor	PV of Notional Principal
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1/06-6/06	1	180	4.00%	2.000%	$100,000,000	0.9804	$49,020,000
7/06-12/06	2	180	4.25%	2.125%	$100,000,000	0.9600	$48,000,000
1/07-6/07	3	180	4.50%	2.250%	$100,000,000	0.9389	$46,945,000
7/07-12/07	4	180	4.75%	2.375%	$100,000,000	0.9171	$45,855,000
1/08-6/08	5	180	5.00%	2.500%	$100,000,000	0.8947	$44,735,000
7/08-12/08	6	180	5.25%	2.625%	$100,000,000	0.8718	$43,590,000
						PV of Notional Principal=	$278,145,000

Column Description
A= Period the interest rate is in effect
B= Period number (t)
C= Number of days in the period (semi-annual=180 days)
D= Annual interest rate for the future period from financial publications
E= Semi-annual rate for the future period (D/2)
F= Notional principal from swap contract
G= Discount factor=1/[(forward rate for period 1)(forward rate for period 2)...(forward rate for period t)]
H= PV of notional principal [F × (C/360) × G]

In the third step, the results from the first steps are used to solve the theoretical swap rate:

$$\text{Theoretical Swap Rate} = \frac{\$12,816,663}{\$278,145,000} = 4.61\%$$

Hence, the issuer (fixed-rate payer) will be willing to pay a fixed 4.61 percent rate for the life of the swap contract in return for receiving 6-month LIBOR.

In the final step, the swap spread is calculated. The market convention is to use a U.S. Treasury security of comparable maturity as a benchmark. For example, if a three-year U.S. Treasury note had a yield to maturity of 4.31 percent, the swap spread in this case would be 30 basis points (4.61% - 4.31% = 0.30%).

2.4.2.2 Currency Swaps

A currency swap involves exchanging principal and interest payments in one currency for principal and interest payments in another currency. A currency swap requires that the principle is specified in each of the two currencies. These principles are then exchanged at the beginning and end of the currency swap using the exchange rates to make each payment an equivalent amount.

The value of a swap where a bond in U.S. dollars (B_D) is received and a bond in a foreign currency (B_F) is paid is valued as:

$$V_{swap} = B_D - S_0 B_F$$

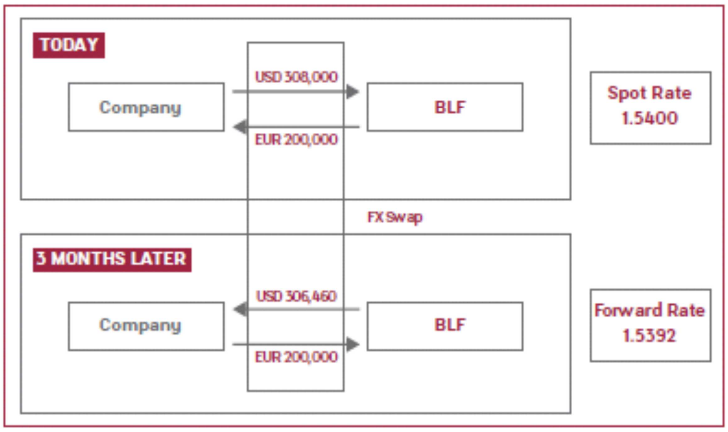
Similarly, the value of a swap where a bond in a foreign currency (B_F) is received and a bond in U.S. dollars (B_D) is paid is valued as:

$$V_{swap} = S_0 B_F - B_D$$

Where S_0 is the spot exchange rate.

Worked Example:[37]

Suppose a company has dollars available today and is aware that it will need them in three months. It also requires EUR 200,000 for its daily needs. The company can choose either to sell dollars for euros and, three months later to pay back dollars by selling those euros, or to make a deposit in dollars with the bank and to take a loan in euros from that bank. However, the best solution may be a three-month currency swap transaction whereby the company buys EUR 200,000 needed for its immediate use and sells its dollars on the spot market, while simultaneously selling those EUR 200,000, three months later (when the company needs its dollars back) on the forward market, according to the currency exchange rate agreed upon on the day the transaction is booked. In this way, the currency swap transaction would be used to regulate money flows while eliminating foreign exchange risks and/or avoiding loans.



[37] Example sourced from "Currency Swap," *Capital Letter*, Issue 3, Banque Libano-Française Finance.

2.4.2.3 *Credit Default Swaps*[38]

A credit default swap (CDS) insures against the risk of default by a particular company. The company being insured is called the *reference entity* and default by the reference entity is called a *credit event*. The buyer of the insurance obtains the right to sell a particular bond (*reference obligation*) issued by the company at par value (*notional principle*) if a credit event occurs.

The buyer of a CDS makes periodic payments to the seller until the end of the CDS or until a credit event occurs. A credit event requires that the buyer makes a final accrual payment, either in physical delivery or cash as settlement.

The annual payment as a percentage of the notional principle for a newly issued CDS is the *CDS spread* (*s*), which is valued based on the following: [39]

$$s = \frac{\int_0^T \left[1 - R - A(t)R\right] q(t) v(t)\, dt}{\int_0^T a(t)\left[u(t) + e(t)\right] dt + \pi u(t)}$$

Where:

T	is the life of the CDS in years
p_i	is the risk-neutral probability of default at time t_i
$\hat{R}$	is the expected recovery rate on the reference obligation in a risk-neutral world
u(t)	is the present value of payments at an annual rate
e(t)	is the present value of a payment at time *t* equal to *t* - *t** (where *t** is the payment date immediately preceding time *t*)
v(t)	is the present value of the notional principle at time *t*
w	is the payments per year made by the CDS buyer
s	is the value of *w* that leads to the CDS having a value of zero
π	is the risk-neutral probability that there will be no credit event during the life of the CDS
A(t)	is the accrued interest on the reference obligation at time *t* as a percentage of the face value

[38] Ibid, Chapter 27.

[39] Moorad Choudhry, M. (2010), *Structured Credit Products: Credit Derivatives and Synthetic Securitisation*, 2nd ed., John Wiley & Sons, Singapore, Pgs. 253-254.

2.4.2.4 Commodity Swaps

A commodity swap is an agreement to exchange a fixed amount of a commodity at fixed times in the future at a fixed price. Commodity swaps are the most recent types of swaps and the majority involve crude oil.

Worked Example: [40]

An Oil Producer of 300,000 bbl/month sells crude oil to its customers at an agreed-upon index price. The firm wants more predictable cash flows in order to determine its ability to capitalize on exploration and production opportunities next year. To help accomplish this objective, the Producer enters into a one-year swap with SET to hedge 1/3 of its production at a fixed price of $22.00/bbl. This swap hedge is financially equivalent to a forward sale of 100,000 barrels of crude oil per month for 12 months. On each Settlement Date, the Producer receives from SET a fixed payment equal to $22.00/barrel. The Producer, in exchange, makes a floating payment to SET based on the arithmetic average of the daily settlement prices of the prompt NYMEX crude oil futures contract for each of the Pricing Periods for which the Reference Price is quoted. The floating payment paid to SET should closely approximate the payment received by the Producer from its customer(s) for physical deliveries of crude oil. The net result is that by combining the swap with its current physical crude oil contract, the Producer receives $22.00/bbl for its oil sales. Hence, the sample terms are as follows:

Description	:	Fixed for Floating Crude Oil Swap Contract
Contract Maturity	:	One year
Fixed Price Payer	:	Sempra Energy Trading Corp.
Floating Price Payer	:	The Crude Oil Producer
Settlement Type	:	Financial
Settlement Dates	:	Five business days after the last day of the Pricing Period, for a total of 12 Settlement Dates
Pricing Periods	:	Twelve full calendar months from July 2000 through June 2001
Reference Quantity	:	100,000 barrels per month
Reference Price	:	The daily Official Settlement price of the prompt NYMEX WTI Futures Contract in $/bbl
Floating Price	:	The arithmetic average of the Reference Price during the Pricing Period rounded to the nearest $ 0.01 / bbl
Floating Payment	:	Floating Price * Reference Quantity
Fixed Price	:	$ 22.00 per barrel
Fixed Payment	:	Fixed Price * Reference Quantity
Documentation	:	Sempra Energy Trading Corp. Standard Swap Agreement
Credit Arrangements	:	To be determined

[40] Example sourced from http://www.sempracommodities.com/oil_producers.asp

Thus the swap transaction results given different monthly average WTI prices on a Settlement Date in US$/bbl for the first 5 months are as follows:

WTI Monthly Average Price	17.50	19.50	21.50	23.50	25.50
Swap Cash Flows :					
Swap Fixed Payment (Received)	22.00	22.00	22.00	22.00	22.00
Floating Payment (Paid)	-17.50	-19.50	-21.50	-23.50	-25.50
Swap Result	4.50	2.50	0.50	-1.50	-3.50

2.5 Swaptions

2.5.1 Definitions and Concepts

A swaption is an over-the-counter option on a swap arrangement and thus grants the owner the right but not the obligation to enter into an underlying swap. The advantage of a swaption is that unlike ordinary swaps, a swaption hedges the buyer against downside risk and also enables the buyer to take advantage of any upside benefits. Although swaptions can be traded on a variety of swaps, the most common type of swaptions are options on interest rate swaps. Companies often use swaptions to ensure that a fixed interest rate that is payable at some future date does not exceed a pre-specified level and is thus used to hedge interest rate risk.

a. **Payer swaption** - gives the owner of the swaption the right but not the obligation to enter into a swap that pays the fixed leg and receives the floating leg.
b. **Receiver swaption** - gives the owner of the swaption the right but not the obligation to enter into a swap that receives the fixed leg and pays the floating leg.
c. **American swaption** - where the owner is allowed to enter the swap on any day that falls within a range of two dates.
d. **European swaption** – where the owner is allowed to enter the swap only on the maturity date.

2.5.2 Hedging with Swaptions[41]

Two common forms of hedging with swaptions are delta-hedging and delta-gamma-hedging.

2.5.2.1 Delta-Hedging

Delta-hedging is a dynamic hedging strategy where the price changes of a swap are compensated for with price changes of a swaption. Delta-hedging is achieved by holding (or shorting) a swaption and shorting (or holding) a quantity of the underlying (swap). This combination is referred to as a hedge portfolio. Thus, price increases of the swap are compensated by price drops of the swaption and vice-versa. Consequently, fluctuations of the underlying security are almost eliminated.

[41] Akume, D., Luderer, B. and Weber, G.W. (2003), "Pricing and Hedging of Swaptions," *Tom*, Vol. 8, No. 4.

2.5.2.2 Gamma-Neutral

The frequent hedging associated with delta-hedging can be expensive and thus it is natural to try to minimize the need to rebalance the portfolio too frequently. The hedging technique that seeks to rebalance the portfolio only at specific points in time is called a gamma-neutral strategy. This hedge is achieved by buying and selling enough swaptions to make the portfolio both delta- and gamma-neutral.

2.5.3 Valuing Swaptions[42]

The standard approach to valuing European swaptions is to use the *Black model*, which assumes that the relevant swap rate is at the maturity of the option and is lognormal. Thus the values of swaptions are given by the following equations:

$$P = \left[\frac{1 - \frac{1}{\left(1 + \frac{F}{m}\right)^{t_1 m}}}{F} \right] e^{-rT} [FN(d_1) - XN(d_2)]$$

$$R = \left[\frac{1 - \frac{1}{\left(1 + \frac{F}{m}\right)^{t_1 m}}}{F} \right] e^{-rT} [XN(-d_2) - FN(-d_1)]$$

Where:

P is the payer swaption

R is the receiver swaption

t is the tenor of a swap in years

F is the forward rate of an underlying asset swap

X is the strike rate of a swaption

r is the risk-free interest rate

[42] http://www.riskworx.com/pdfs/instruments/swaptions.pdf

T is the time to expiration in years

s is the volatility of the forward-starting swap rate

m is the compoundings per year in swap rate

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

Worked Example:

Consider a 2-year payer swaption on a 4-year swap with semi-annual compounding. The forward swap rate of 7% starts 2 years from now and ends 6 years from now. The strike is 7.5%; the risk-free interest rate is 6%; the volatility of the forward starting swap rate is 20% p.a. What is the value of the payer option?

$$d_1 = \frac{\ln\left(\frac{0.07}{0.075}\right) + \left(\frac{0.2^2}{2}\right)2}{0.2\sqrt{2}} = -0.1025 \text{ and } d_2 = -0.1025 - 0.2\sqrt{2} = -0.38535$$

$N(d_1) = 0.4592$ and $N(d_2) = 0.35$

Hence the value of the payer swaption is 1.8% of the notional calculated as follows:

$$P = \left[\frac{1 - \frac{1}{\left(1 + \frac{0.07}{2}\right)^{4\times 2}}}{}\right] e^{-0.06\times 2}[0.07(0.4592) - 0.075(0.35)] = 0.017967$$

CHAPTER 3: MARKET RISK

Recommended Reading: Textbook Chapters 7, 9 and 13

3.1 Definitions and Concepts

Market risk is the risk to a financial portfolio arising from changes in market conditions. Market risks are further broken down into the components of equity risks, exchange rate risks, interest rate risks and commodity price risks.

a. **Equity risk** – the risk that an investment will lose value arising from movements in equity markets.

b. **Exchange rate risk** – the risk that an investment will lose value arising from movements in exchange rates.

c. **Interest rate risk** – the risk that an interest-bearing investment will lose value due to changes in interest rates.

d. **Commodity risk** - the risk that an investment will lose value arising from movements in commodity prices.

3.2 Measuring Market Risk[43]

Value-at-Risk (VaR) emerged after the 1987 stock market crash. At that time, many physicists had moved from scientific academia and research into corporations where they could design complex financial instruments and hedging models. Thus the stock market crash raised doubts about the robustness of the existing models when faced with an extreme event. VaR was then developed to segregate the extreme events from everyday market movements and thus provide an 'early warning' system. Risk management VaR was further developed in the 1990s by the JP Morgan CEO's (Dennis Weatherstone) "4:15 report," which combined all of the risks that the firm faced on a single page, available within 15 minutes of the market close. In 1994 VaR began to be more widely used after JP Morgan published the methodology and provided access to the underlying parameters (in 1996, JP Morgan developed the approach into a separate business, which became RiskMetrics). In 1997, following a series of derivative related financial disruptions such as the Barings Bank collapse; the U.S. SEC pronounced that public companies would need to disclose quantitative information with regard to their derivative instruments. Hence, most major banks and financial institutions started including VaR disclosures in the notes to their financial statements.[44] In 1999 Basel II was adopted globally and the use of VaR as a measure of market risk became standard.

[43] Dowd, K. (2002), *An Introduction to Market Risk Measurement*, John Wiley & Sons, UK.
[44] Jorion, P. (2006), *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd ed., McGraw-Hill.

VaR models seek to provide a single number that summarizes the total risk exposure of a financial instrument. The VaR metric has two characteristics:

a. It provides a common consistent measure of risk across different positions and risk factors; and
b. It takes account of the correlations between different risk factors.

In practice, VaR is used in the following ways:

- To set a risk target or boundary.
- To determine capital requirements (the riskier the activity, the higher the VaR and the greater the capital requirement).
- For disclosing financial instruments and risk.
- To assess the risks of different investment opportunities before decisions are made.
- To implement portfolio-wide hedging strategies.
- To discourage 'moral hazard' excessive risk-taking.

3.2.1 The VaR Measure[45]

VaR is typically used to give a measure of the financial risk exposure facing a financial institution within a specified percentage of certainty over a specific period of time. Thus it is a function of two parameters: the time horizon (N) and the confidence level (X). Hence it is the loss level over N days that a portfolio manager is X% certain will not be exceeded. In the banking sector, regulators often set N=10 and X=99 and thus focus on the loss level over a 10-day period that is expected to be exceeded by only 1% of the time. The bank's capital requirement is then set at three times this VaR measure. In general, VaR is the loss corresponding to the (100 - X)th percentile of the normal distribution of the change in the value of the portfolio over N days.

3.2.2 Traditional Approaches[46]

3.2.2.1 Gap Analysis

Gap analysis was developed by financial institutions in order to derive a simple measure of interest-rate risk exposure. The approach starts by choosing a horizon period and then determining by how much an asset or liability portfolio will re-price within this period and thus provides an indication of the sensitivity of the portfolio. Hence, the interest-rate exposure of the portfolio is deemed to be the change in net interest income that arises from changes in interest rates, measured as:

$$\Delta NII = (GAP)\Delta r$$

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[45] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pgs. 346-348.

[46] Dowd, K. (2002), *An Introduction to Market Risk Measurement*, John Wiley & Sons, UK.

Where ΔNII is the change in net interest income and Δr is the change in interest rates. Although gap analysis is simpler to undertake, the approach has limitations:

- It only applies to on-balance sheet interest-rate risk;
- It only looks at the impact of interest rates on income (rather than on asset or liability values); and
- Results can be sensitive to the choice of horizon period.

3.2.2.2 Duration Analysis

In addition to gap analysis, another approach that is often used to measure interest rate risk is duration analysis. The duration D of a fixed-income asset (such as a bond) is defined as the weighted average term to maturity of the asset's cash flows, where the weights are the present values of each cash flow relative to the present value of all cash flows. Thus duration can be measured as:

$$D = \frac{\sum_{i=1}^{n}[i \times PVCF_i]}{\sum_{i=1}^{n} PVCF_i}$$

where $PVCF_i$ is the present value of the cash flow in period i, discounted at the spot period yield. The duration measure D can also provide an indication of a bond's price sensitivity to a change in yield where:

$$(\Delta bond\ price)\% \approx -D \times \frac{\Delta y}{(1+y)}$$

and where y is the yield and Δy the change in yield.

Hence, the bigger the duration, the more the bond price changes in response to a change in yield. The advantages of the duration approach are:

- It is easy to use because duration measures are simple to calculate and the information required is easily obtainable; and
- It is also better than gap analysis because it measures changes in asset (or liability) values rather than just changes in net income.

However, the duration approach has the following limitations:

- It only considers interest-rate risk;
- It is a crude measure that is less accurate than more modern approaches; and
- It has become outdated since the use of computerised risk modelling.

3.2.2.3 Scenario Analysis

Scenario analysis (or 'what if' analysis) uses a set of scenarios to investigate the potential gain or loss that will result if each of the events occur. 'Scenarios' refer to the combination of facts, conditions, and related factors that will be used to produce a finite set of plausible dynamics. The combination of scenario analysis and systems thinking is sometimes called *structural dynamics* and produces plausible solutions that link the causality between assorted factors. Unfortunately, scenario analysis is not easy to undertake since the outcome of the analysis is reliant on the accuracy and completeness of the scenarios. Thus, scenario analysis is largely subjective and many elements of the process are qualitative rather than quantitative.

3.2.2.4 Portfolio Theory[47]

Portfolio theory was introduced in the work of Markowitz (1952, 1959).[48] Portfolio theory is based on the basic premise that investors choose between portfolios on the basis of their expected return and the portfolio risk. The standard deviation of the portfolio return is typically regarded as a measure of the portfolio's risk. Thus, an investor wants a portfolio with a high return but a low standard deviation. This can be achieved by choosing a portfolio that maximises the expected return for a portfolio's standard deviation or minimises the standard deviation for an expected return. A portfolio that meets these conditions is deemed to be efficient and will thus be chosen by a rational investor. An investor must make an investment decision by designing a set of efficient portfolios. However, different efficient portfolios will have different risk profiles and thus the investor must choose a portfolio on the basis of his personal trade-off between risk and expected return. For example, a risk-averse investor will choose a safe portfolio with a low standard deviation and a low expected return, while an investor with a greater risk appetite will choose a portfolio with a higher expected return and a higher standard deviation.

A fundamental insight of portfolio theory is that the risk of any individual asset is the extent to which that asset contributes to the risk profile of the portfolio rather than just the standard deviation of the individual asset. An individual asset may have a high standard deviation but could have a return that is correlated with the returns of the other assets in a portfolio such that the inclusion of the additional asset does not increase the standard deviation of the portfolio. Consequently, the inclusion of this asset would be risk-free even though the asset is risky when considered on its own. Thus another fundamental insight of portfolio theory is that investors can reduce their risk exposure

[47] Damodaran, A. (2002), *Investment Valuation*, 2nd ed., John Wiley & Sons, New York.
[48] Markowitz, H.M. (1952), "Portfolio Selection," *Journal of Finance*, Vol. 7, Pgs. 77–91.
Markowitz, H.M. (1959), *Portfolio Selection: Efficient Diversification of Investments*, John Wiley & Sons, New York.

to an individual asset by holding a diversified portfolio of assets. The basic component equations used in portfolio theory are:

a. **Expected return**

$$E(R_p) = \sum_i w_i E(R_i)$$

b. **Portfolio return variance**

$$\sigma_p^2 = \sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_{j \neq 1} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

c. **Portfolio return for a two asset portfolio**

$$E(R_p) = \omega_A E(R_A) + \omega_B E(R_B) = \omega_A E(R_A) + (1 - \omega_A) E(R_B)$$

d. **Portfolio variance for a two asset portfolio**

$$\sigma_p^2 = \omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\omega_A \omega_B \sigma_A \sigma_B \rho_{AB}$$

e. **Portfolio return for a three asset portfolio**

$$E(R_p) = \omega_A E(R_A) + \omega_B E(R_B) + \omega_C E(R_C)$$

f. **Portfolio variance for a three asset portfolio**

$$\sigma_p^2 = \omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + \omega_C^2 \sigma_C^2 + 2\omega_A \omega_B \sigma_A \sigma_B \rho_{AB} + 2\omega_A \omega_C \sigma_A \sigma_C \rho_{AC} + 2\omega_B \omega_C \sigma_B \sigma_C \rho_{BC}$$

Where:

R_p is the return on the portfolio

R_i is the return on asset *i*

ω_i is the proportion of asset *i* in the portfolio

σ^2 is the variance

σ is the standard deviation

ρ_{ij} is the correlation coefficient between the returns on assets *i* and *j*

In addition, the extent to which a new asset contributes to the risk of a portfolio depends on the correlation or covariance of its return in relation to the returns of the other assets in the portfolio. The measure typically used to capture this relationship is beta, which is measured as:

$$\beta_i = \frac{\text{Covariance of asset } i \text{ with market portfolio}}{\text{Variance of the market portfolio}} = \frac{\sigma_{im}}{\sigma_m^2}$$

The covariance of the market portfolio with itself is its variance and thus the beta of the market portfolio is 1. Assets that are riskier than the market will have a beta that is greater than 1 and assets that are less risky than the market will have betas less than 1.

However, portfolio theory has been criticized as being based on unrealistic assumptions:

- Asset returns are assumed to be normally distributed thus precluding the skewness and kurtosis associated with 'fat tails';
- The correlations between the assets are assumed to be constant;
- It is assumed that investors seek to maximize their economic utility, and are rational and risk-averse (in accordance with the *efficient market hypothesis*);
- It is assumed that all investors have access to the same information at the same time (in accordance with the efficient market hypothesis);
- It is assumed that investors do not influence prices and have no credit limits;
- Taxes and transaction costs are excluded; and
- Models market expectations rather than the market itself.

3.2.3 Volatility Measurement[49]

3.2.3.1 RiskMetrics Variance Model

JP Morgan's RiskMetrics variance model (also known as exponential smoother) consists of the following equation:

$$\sigma_{t+1}^2 = (1-\lambda)\sum_{\tau=1}^{\infty} \lambda^{\tau-1} R_{t+1-\tau}^2 \quad (\text{for } 0 < \lambda < 1)$$

Hence by applying exponential smoothing, today's variance can be defined as:

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[49] Christoffersen, P.F. (2003), *Elements of Financial Risk Management*, Chapter 2, Academic Press, Elsevier Science, USA.

$$\sigma_t^2 = (1-\lambda)\sum_{\tau=1}^{\infty}\lambda^{\tau-1}R_{t-\tau}^2 = \frac{1}{\lambda}(1-\lambda)\sum_{\tau=2}^{\infty}\lambda^{\tau-1}R_{t+1-\tau}^2$$

And thus tomorrow's variance can be defined as:

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)R_t^2$$

Where:

R is the assets log-return

τ is a small increment of time t

λ is the smoothing parameter

The model thus shows that recent returns are more significant when forecasting future variance than distant returns as λ gets smaller as τ gets larger. Research conducted by RiskMetrics found that across various assets it is possible to set λ=0.94. However, the RiskMetric model is subject to certain limitations since it ignores the relative stability of long-run average variance. Thus if variance today is high, then according to the RiskMetrics model, all future variance will be high as well.

3.2.3.2 GARCH Models

The Generalised Autoegressive Conditional Heteroscedastic (GARCH) model was developed by Bollerslev (1986)[50] and Taylor (1986).[51] The simplest GARCH (1,1) model can be described by the equation:

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \beta\sigma_t^2 \qquad \text{(with } \alpha + \beta < 1)$$

Thus the RiskMetrics variance model is a form of GARCH model where $\alpha = 1 - \lambda$, $\beta = \lambda$, $\alpha + \beta = 1$, and $\omega = 0$. However, an important difference between the RiskMetrics and GARCH model is that the GARCH model relies on σ^2, which is defined as:

$$\sigma^2 = \frac{\omega}{(1-\alpha-\beta)}$$

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[50] Bollerslev, T. (1986), "Generalised Autoregressive Conditional Heteroscedasticity," *Journal of Econometrics*, Vol. 31, Pgs. 307-327.

[51] Taylor, S.J. (1986), "Forecasting the Volatility of Currency Exchange Rates," *International Journal of Forecasting*, Vol. 3, Pgs. 159-170.

Since $\alpha + \beta$ measures the persistence of variance, if:

- $\alpha + \beta = 1$ then shocks to conditional variance are highly persistent.
- $\alpha + \beta = < 1$ then shocks to conditional variance are mean reverting.
- $\alpha + \beta = > 1$ then shocks to conditional variance are explosive.

The GARCH model thus assumes that future variance will revert to its average value ($\alpha + \beta = < 1$) while the RiskMetrics model assumes that shocks are highly persistent ($\alpha + \beta = 1$).

There are numerous extensions of the basic GARCH model including:

a. **GJR models** – developed by Glostern, Jagannathan, and Runkel (1993)[52] and includes an additional term to take account of the leverage effect, which occurs where a negative return increases the variance by more than a positive return:

$$\sigma_{t+1}^2 = \omega + \alpha_1 R_t^2 + \alpha\theta I_t R_t^2 + \beta\sigma_t^2 \quad \text{where} \quad I_t = 1 \text{ if } R_t < 0$$
$$I_t = 0 \text{ otherwise}$$

Thus there will be a leverage effect if $\theta > 0$.

b. **EGARCH models** - exponential GARCH (EGARCH) model assumes that the leverage effect is exponential. EGARCH was proposed by Nelson (1991)[53] and can be specified as:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{R_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|R_t|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

Thus the EGARCH has two primary advantages over the standard GARCH model:

- There is no need to impose non-negativity constraints on the model since $\ln(\sigma_t^2)$ will be positive; and
- Asymmetries are possible since if the relationship between volatilities and returns are negative then γ will be negative.

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[52] Glostern, L.R., Jagannathan, R. and Runkel, D.E. (1993), On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," *The Journal of Finance*, Vol. 48, No. 5, Pgs. 1779-1801.
[53] Nelson, D.B. (1991), "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Econometrica*, Vol. 59, No. 2, Pgs. 347-370.

c. **CGARCH models** – the component GARCH (CGARCH) model of Engle and Lee (1999)[54] separate long-run and short-run volatility. Thus, while the GARCH model demonstrates mean reversion in volatility to ω, the CGARCH model allows mean reversion to a time-varying, q_t. The CGARCH model can be specified as:

$$\sigma_{t+1}^2 = q_{t+1} + \alpha(R_t^2 - q_t) + \beta(\sigma_t^2 - q_t) \quad \text{and} \quad q_{t+1} = \omega + \rho q_t + \phi(R_t^2 - \sigma_t^2)$$

Where:

q_t	is long-run (or trend) volatility provided $\rho > (\alpha + \beta)$
$(R_t^2 - \sigma_t^2)$	is the forecasting error (which is the driver for the time-dependent movement of the trend)
$(R_t^2 - q_t)$	is the transitory component of the conditional variance

The transitory component will converge to zero with powers of α + β while the long-run component will converge to q_t with powers of ρ if stationarity is achieved, which occurs when $(\alpha + \beta)(1 - \rho) + \rho < 1$ and thus $\rho < 1$ and $(\alpha + \beta) < 1$.

Worked Example:[55]

Use a GARCH(1,1) model to estimate the 1 percent value at risk of a $1,000,000 portfolio on March 23, 2000. This portfolio consists of 50 percent Nasdaq, 30 percent Dow Jones and 20 percent long bonds. The long bond is a ten-year constant maturity Treasury bond.

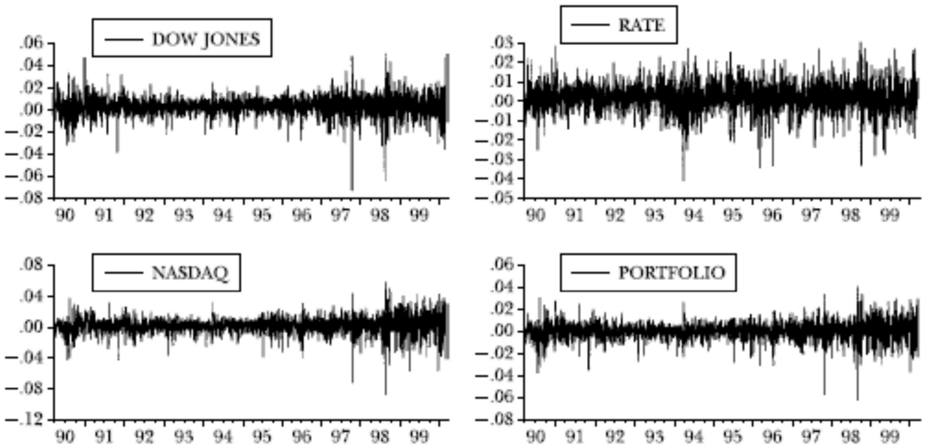
In the first step, a hypothetical historical portfolio is constructed. The figure shows the pattern of returns of the Nasdaq, Dow Jones, bonds, and the composite portfolio leading up to the terminal date. Each of these series appears to show the signs of ARCH effects in that the amplitude of the returns varies over time. In the case of the equities, it is clear that this has increased substantially in the latter part of the sample period. Visually, Nasdaq is even more extreme.

The table presents some illustrative statistics for each of these three investments separately and for the portfolio as a whole in the final column. From the daily standard deviation, we see that the

[54] Engle, R.F. and Lee, G.G.J. (1999), "A Permanent and Transitory Component Model of Stock Return Volatility," in Engle, R.F. and White, H. (eds.), *Cointegration, Causality and Forecasting: A Festschrift in Honour of Clive W.J. Granger*, Oxford University Press, Oxford.

[55] Example sourced from Engle, R. (2001), "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics," *Journal of Economic Perspectives*, Vol. 15, No. 4, Pgs. Pages 157–168.

Nasdaq is the most volatile and interest rates the least volatile of the assets. The portfolio is less volatile than either of the equity series even though it is 80 percent equity—yet another illustration of the benefits of diversification. All the assets show evidence of fat tails, since the kurtosis exceeds 3, which is the normal value, and evidence of negative skewness, which means that the left tail is particularly extreme.



	NASDAQ	*Dow Jones*	*Rate*	*Portfolio*
Mean	0.0009	0.0005	0.0001	0.0007
Std. Dev.	0.0115	0.0090	0.0073	0.0083
Skewness	−0.5310	−0.3593	−0.2031	−0.4738
Kurtosis	7.4936	8.3288	4.9579	7.0026

Sample: March 23, 1990 to March 23, 2000.

In the next step, the standard deviation of the portfolio and its 1 percent quantile are forecasted over several different time frames: the entire ten years of the sample up to March 23, 2000; the year before March 23, 2000; and from January 1, 2000, to March 23, 2000. Consider first the quantiles of the historical portfolio at these three different time horizons. To do this calculation, one simply sorts the returns and finds the 1 percent worst case. Over the full ten-year sample, the 1 percent quantile times $1,000,000 produces a value at risk of $22,477. Over the last year, the calculation produces a value at risk of $24,653—somewhat higher, but not enormously so. However, if the 1 percent quantile is calculated based on the data from January 1, 2000, to March 23, 2000, the value at risk is $35,159. Thus, the level of risk apparently has increased dramatically over the last quarter of the sample. Each of these numbers is the appropriate value at risk if the next day is equally likely to be the same as the days in the given sample period. This assumption is more likely to be true for the shorter period than for the long one.

The basic GARCH(1,1) results are given in the table. Under this table it lists the dependent variable, PORT, and the sample period, indicates that it took the algorithm 16 iterations to maximize the likelihood function and computed standard errors using the robust method of Bollerslev-Wooldridge. The three coefficients in the variance equation are listed as C, the intercept; ARCH(1), the first lag of the squared return; and GARCH(1), the first lag of the conditional variance. Notice that the coefficients sum up to a number less than one, which is required to have a mean reverting variance process. Since the sum is very close to one, this *process only mean* reverts slowly. Standard errors, Z-statistics (which are the ratio of coefficients and standard errors) and p-values complete the table.

	Variance Equation			
Variable	*Coef*	*St. Err*	*Z-Stat*	*P-Value*
C	1.40E-06	4.48E-07	3.1210	0.0018
ARCH(1)	0.0772	0.0179	4.3046	0.0000
GARCH(1)	0.9046	0.0196	46.1474	0.0000

Notes: Dependent Variable: PORT.
Sample (adjusted): March 23, 1990 to March 23, 2000.
Convergence achieved after 16 iterations.
Bollerslev-Woodridge robust standard errors and covariance.

The forecast standard deviation for the next day is 0.0146, which is almost double the average standard deviation of 0.0083 presented in the last column of the first table. If the residuals were normally distributed, then this would be multiplied by 2.327, because 1 percent of a normal random variable lies 2.327 standard deviations below the mean. The estimated normal value at risk is \$33,977. As it turns out, the standardized residuals are not very close to a normal distribution. They have a 1 percent quantile of 2.844, which reflects the fat tails of the asset price distribution. Based on the actual distribution, the estimated 1 percent value at risk is \$39,996.

3.2.4 Simulation Models

3.2.4.1 Historical Simulation (HS)

Historical simulation uses a sample of past returns to model future returns. Thus the HS approach extends the standard definition of portfolio returns as follows:

$$R_{p,t+1} = \sum_{i=1}^{n} \omega_i R_{i,t+1} \Rightarrow \left\{ R_{p,t+1-\tau} \right\}_{\tau=1}^{m} \equiv \left\{ \sum_{i=1}^{n} \omega_i R_{i,t+1-\tau} \right\}_{\tau=1}^{m}$$

Where:

R_p is the return of the portfolio

ω_i is the proportion of asset *i* in the portfolio

R_i is the return on asset *i*

τ is a small increment of time *t*

m is a sample of historic daily returns (typically chosen between 250 – 1000 days)

Hence HS assumes that it is possible to approximate tomorrow's portfolio returns $(R_{p,t+1})$ using the distribution of the past *m* observations $\{R_{p,t+1-\tau}\}_{\tau=1}^{m}$. The value at risk (*Va*R) can then be calculated from the HS output by simply sorting the returns in $\{R_{p,t+1-\tau}\}_{\tau=1}^{m}$ in ascending order and selecting the *Va*R where a certain percentage of the number of observations is lower than the specified *Va*R. The two main advantages of using HS are:

- The approach is simple to use; and
- It does not assume any distribution on the asset returns.

However, HS also has five main short-comings:

- Estimating the data sample length of *m* is difficult. If m is too small then there will not be enough data to calculate *Va*R, and if *m* is too large then the *Va*R will not be sufficiently responsive to the more recent observations;
- HS is slow to adapt to changing market conditions;
- HS assumes that asset returns are *iid* (independent and identically distributed), which is unrealistic as asset returns commonly exhibit volatility clustering;
- HS applies equal weight to all returns of the whole period; and
- HS assumes constant volatility and covariance of the raw returns and thus does not reflect market changes.

3.2.4.2 Weighted Historical Simulation (WHS)

Weighted historical simulation compensates for the difficulties associated with *m* by assigning greater weight to the most recent observations and less weight to more distant observations. WHS consist of a three-step process:

a. The sample of *m* past returns is assigned a probability weighting that declines exponentially from present to past as follows:

$$\eta_\tau = \left\{ \frac{\eta^{\tau-1}(1-\eta)}{(1-\eta^{m})} \right\}_{\tau=1}^{m}$$

(Note that today's observation is η_1; as τ gets larger, η_τ tends to zero; the weights from $\tau = 1, 2, ..., m$ sum to 1; and η typically lies between 0.95 and 0.99).

b. The observations and their assigned weights are sorted into ascending order;

c. The VaR is calculated by accumulating the weights of the ascending returns until the critical percentage is reached.

The advantages of WHS are:

- It is easy to implement once η has been selected;
- Weighting lessens the potentially detrimental impacts of m; and
- Builds conditionality into the process whereby recent market conditions are more important.

Unfortunately, WHS also has short-comings:

- There are no guidelines on how to assign η;
- Weighting can lead WHS to assign disproportionate impacts between positive and negative values;
- As in the case of HS, WHS also requires a large amount of historic data in order to calculate the *VaR*.

3.2.4.3 Filtered Historical Simulation (FHS)

Filtered historical simulation (FHS) was proposed by Barone-Adesi, Giannopoulos, and Vosper (1999)[56] and is a Monte Carlo simulation technique that makes it possible to use the historic returns to define the distribution rather than having to assume that the distribution is *iid* (as in HS and WHS). In order to make the distributions *iid*, volatility clustering is removed by modeling the returns in a GARCH model and serial correlation is removed using a moving-average (MA) term in the GARCH conditional mean equation. FHS is best used to calculate a 10-day, 1% *VaR*[57] and consists of the following steps:

a. Use a sequence of historic returns $\left\{R_{p,t+1-\tau}\right\}_{\tau=1}^{m}$ to estimate a GARCH model;

b. Calculate the past standardised returns from the observed returns and from the estimated standard deviations $\hat{z}_{t+1-\tau} = \left(\frac{R_{t+1-\tau}}{\sigma_{t+1-\tau}}\right)$ and produce the set of standardised returns $\left\{\hat{z}_{t+1-\tau}\right\}_{\tau=1}^{m}$

c. Build up a distribution of future returns using the set of standardised returns instead of drawing $\hat{z}$ from a random number generator.

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56 Barone-Adesi, G., Giannopoulos, K. and Vosper, L. (1999), "VaR Without Correlations for Portfolio of Derivative Securities," *Journal of Futures Markets*, Vol. 19, Pgs. 583-602.

57 Piroozfar, G. (2009: 9), "Forecasting Value at Risk with Historical and Filtered Historical Simulation Methods," Department of Mathematics, Uppsala University, *U.U.D.M. Project Report 2009:15*.

d. Collect the sequences into a set of K-day ahead returns $\left\{\hat{R}_{i,t+1:t+K}\right\}_{i=1}^{F}$ (where F is the number of times that the standardised residuals are drawn on each future date and K is the time horizon measured in days).

e. Calculate the VaR: $VaR_{t+1:t+K} = -Percentile\left[\left\{\hat{R}_{i,t+1:t+K}\right\}_{i=1}^{F}, 100\right]$

3.2.4.4 Monte Carlo Simulation (MCS)[58]

Monte Carlo methods use random sampling to simulate the impacts of various sources of uncertainty and then determine the average over the range of resultant outcomes. The simulations relate to an algorithm that generates random numbers, yielding an approximation to a closed-form formula. MCS typically makes use of a five-step process:

a. The length (T) of the MC time horizon is divided into a large number (N) of increments (Δt).

b. The price of the asset at the end of the first time increment is updated using a random number from a random number generator. Thus a share price can be simulated as follows:

$$R_i = (S_{i+1} - S_i) / S_i = \mu \Delta t + \sigma \varphi \sqrt{\Delta t}$$

Where:

R_i	is the return on the ith day
S_i	is the share price on the ith day
S_{i+1}	is the share price on the i+1th day
μ	is the sample mean of the share price $\mu = \frac{1}{T\Delta t}\sum_{i=1}^{T} R_i$
Δt	is the timestep
σ	is the sample volatility of the share price $\sigma = \sqrt{\frac{1}{(T-1)\Delta t}\sum_{i=1}^{T}(R_i - \bar{R})^2}$
φ	is a random number generated from a normal distribution

c. Step 2 is repeated until the end of the analysis horizon (T) is reached.

d. Steps 2 and 3 are repeated for a large number (M) of times to generate many different paths of the share over T. M may be from 1,000 iterations to over 10,000.

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[58] JP Morgan, "An Overview of Value-at-Risk: Part III - Monte Carlo Simulations," available at: http://www.jpmorgan.com/tss/General/Risk_Management/1159380637650 and John C. Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Pgs. 410-414.

e. The M share prices are ranked from the smallest to the largest and the simulated value that corresponds to the desired (1-α)% confidence level (typically 95% or 99%) is isolated to deduce the VaR, which is the difference between S_i and the αth lowest share price.

The advantages of using Monte Carlo simulations compared to other simulation techniques include:

- MCS can model instruments with non-linear and path-dependent payoff functions including complex derivatives;
- MCS is not significantly affected by outliers; and
- MCS can make use of a wide range of possible statistical distributions.

However, MCS also has some disadvantages:

- MCS requires significant software capability
- MCS can be time consuming; and thus
- MCS can be expensive.

Worked Example:[59]

Use a Monte Carlo algorithm to compute the monthly VaR for one stock based on the historical prices as presented below:

In the first step, the sample return mean (-0.17%) and sample return standard deviation (5.51%) are calculated from the series of historical prices. The current price at the end of the 20th of January 2009 was $18.09 but the aim is to compute the monthly VaR on the 20th of January 2009. Hence, the VaR calculation will jump in the future by 22 trading days and look at the estimated prices for

[59] Example sourced from Berry, R. (2009), "An Overview of Value-at-Risk: Part III – Monte Carlo Simulations," *Investment Analytics and Consulting*, March Edition, J.P. Morgan Investment Analytics and Consulting.

the stock on the 19th of February 2009. Since the approach is using the standard stock price model to draw 1,000 paths until T (19th of February 2009), it is necessary to estimate the expected return (also called drift rate) and the volatility of the share on that day. Note that since Δt = 1 day in this example, the expected return (μ) and volatility (σ) measures will equal the sample mean and sample standard deviation. The stock price on S_{i+1} is then calculated from S_i by re-arranging the basic Monte Carlo formula to be:

$S_{i+1} = S_i\left(1 + \mu\Delta t + \sigma\varphi\Delta t\frac{1}{2}\right)$ and then simulating the algorithm with 1,000 paths for the share (a sample of the output is provided below):

	A	B	C	D	E	F
1	READING VaR					
3		Curr. Price	18.09			
4		Percentile	1%			
5		Final Price	15.5730			
6		VaR ($)	-2.5170			
7		VaR (%)	13.91%			
8						
9		Future Prices	Runs	Sorted prices	Runs	
10		17.5762	1	15.1020	201	
11		18.6040	2	15.1069	944	
12		17.5317	3	15.2622	670	
13		19.0869	4	15.2704	357	
14		18.1025	5	15.2780	620	
15		17.9023	6	15.3285	655	
16		18.8282	7	15.3450	999	
17		20.2517	8	15.3893	717	
18		17.3632	9	15.5568	359	
19		17.8109	10	15.5730	398	1%
20		18.8762	11	15.5926	292	
21		16.5372	12	15.6126	150	
22		17.7948	13	15.6734	726	
23		17.6791	14	15.7456	36	
24		18.9303	15	15.8248	62	
25		18.8187	16	15.8326	836	
26		19.6963	17	15.9667	232	
27		17.1252	18	16.0316	95	
28		18.1996	19	16.0590	64	
29		19.4020	20	16.1053	91	2%
30		19.2711	21	16.1208	145	

Thereafter, the 1,000 terminal stock prices are sorted from the lowest to the highest and the price which corresponds to the desired confidence level is identified. Hence, the VaR at a 99% confidence level is identified based on the 1% lowest stock price, which is $15.7530. On January 20th, the stock price was $18.09. Therefore, there is a 1% likelihood that the share falls to $15.7530 or below. If that happens, there will be a loss of at least $18.09 – $15.7530 = $2.5170.

3.2.5 Back-Testing and Stress-Testing[60]

Financial institutions frequently conduct a reality check of their VaR calculations using the techniques of back-testing and/or stress-testing.

3.2.5.1 Back-Testing

Back-testing uses historic data to assess how well a VaR calculation would have performed had it been applied in the past. Thus the back-testing involves replicating historic conditions and then comparing the accuracy of the VaR measure against the back-tested VaR measure. Although back-testing does not predict how a VaR calculation will perform in the future, it does expose the weaknesses since it has been applied to real-world conditions of the past. Back-testing thus allows for testing the accuracy of VaR calculations, as well as facilitating refinement and scenario analysis without having to use actual money. Back-testing is commonly used in VaR calculations that involve asset allocation strategies, share analysis, and trading strategies. However, back-testing is not applicable in cases where there is a feed-back loop.

The common forms of back-testing include:

a. **Unconditional coverage testing** – tests if the fraction of violations obtained for a particular VaR model is significantly different to a hypothesised fraction.
b. **Independence testing** – tests if violations occur in clusters.
c. **Conditional coverage testing** – jointly tests if the VaR violations are independent and the average number of violations is correct.
d. **Shortfall testing** – tests whether the amount predicted by the VaR measure is equal to the expected short-fall risk measure.
e. **Left tail testing** – tests the VaR model's ability to capture the left tail of the distribution where the largest losses will occur.

3.2.5.2 Stress-Testing

Stress testing is used to assess the robustness of a VaR calculation in the event of a major upset (such as a market crash, oil price shock, etc). Stress-testing has become increasingly standard following the adoption of Basel II. Stress-testing gives an indication of how well a VaR calculation will perform should history repeat itself. The three types of scenarios that are usually tested using stress-testing are:

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[60] Christoffersen, P.F. (2003), *Elements of Financial Risk Management*, Chapter 2, Academic Press, Elsevier Science, USA.

a. **Extreme events** – can include the recreation of a historic event, a hypothesized event, and/or a combination of both.
b. **Risk factor shocks** – conducted by shocking any of the VaR calculation's input risk parameters by a user-specified amount.
c. **External factor shocks** – conducted by shocking any of the VaR calculation's parameters with some external shock such as an index shock, macroeconomic shock etc.

3.2.6 Criticism of VaR[61]

Despite its popularity, VaR has been the subject of ongoing criticism, which has increased since the financial crisis of 2008-2009. Although the approach is credited with advancing risk modelling, it is also argued that:

- The transfer of mathematical and statistical models from the physical sciences to social systems (such as financial markets) tends to simplify the dynamic nature of market participants and thus undermines the plausibility of many VaR models, leaving the approach susceptible to major errors.
- VaR estimates are too imprecise to be useful.
- VaR models are exposed to considerable implementation risk whereby different VaR models can produce different VaR estimates.
- VaR contributes a sense of false security.
- When VaR measures are used to control or reward risk taking, traders will have a 'moral hazard' to adjust their risk-taking positions accordingly, which can substantially bias the VaR measures downwards.
- VaR measures are not sub-additive (the VaR of a combined position can be greater than the VaR of the constituent positions).
- There is too much focus on controlling or managing VaR risk measures rather than considering what happens when the VaR measures are exceeded.
- It is assumed that the VaR follows a normal *iid* distribution.
- VaR does not consider systematic risk.

3.2.7 Beyond VaR[62]

Various modified forms of VaR approaches have recently been developed in order to counteract the short-comings of standard VaR techniques, of which the most widely used are incremental-VaR (IVAR) and component-VaR (CVaR).

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[61] Dowd, K. (2002), *An Introduction to Market Risk Measurement*, John Wiley & Sons, UK.
[62] Ibid.

3.2.7.1 Incremental-VaR

Incremental VaR (IVaR) gives us an indication of the extent to which the risks in a VaR calculation change when the VaR portfolio is altered. IVaR is commonly used to assess the impact on VaR of adding a new position to a VaR portfolio. There are three main cases of IVaR:

a. **High IVaR** – occurs when the new position adds substantially to portfolio risk thus negating diversification.
b. **Moderate IVaR** – where the new position adds moderately to portfolio risk.
c. **Negative IVaR** – where the new position reduces overall portfolio risk VaR thus indicating that the new position is a natural hedge against the existing portfolio.

IVaR is typically estimated using two techniques: the before-and-after approach, and Garman's 'delVaR' approach.

a. **Before-and-after analysis** starts with an existing portfolio (*p*) and then maps the portfolio with input market data to obtain a portfolio VaR(*p*). A new portfolio is then constructed that includes some new component (*a*) and the new portfolio VaR is produced VaR(*p* + *a*). The IVaR associated with the new component IVaR(*a*) is then estimated as the difference between the two:

   $$IVaR = VaR(p + a) - VaR(p)$$

   However, the 'before and after' approach is often of limited practical use in many circumstances where there are a large number of different positions as re-evaluating the whole portfolio VaR can be a time-consuming process.

b. **Garman's 'delVaR' approach** was proposed by Garman (1996a-c)[63] and is a method to reduce the computational burden by estimating IVaR using a Taylor-series approximation based on marginal VaRs. The approach begins by mapping *p* and *a* to a set of *n* instruments. If *a* is 'small' relative to *p* then VaR(*p* + *a*) can be approximated by taking a first-order Taylor-series approximation around VaR(*p*):

   $$VaR(p+a) \approx VaR(p) + \sum_{i=1}^{n} \frac{\partial VaR}{\partial \omega_i} \Delta\omega_i$$

   Where ω_i is the size of the mapped position in instrument *i*. The IVaR associated with position *a*, IVaR(*a*), is thus:

[63] Garman, M.B. (1996a), "Making VaR More Flexible," *Derivatives Strategy*, April, Pgs. 52–53; Garman, M.B. (1996b), "Improving on VaR," *Risk*, Vol. 9, May, Pgs. 61–63; Garman, M.B. (1996c), "Making VaR Proactive," Berkeley, CA, *Financial Engineering Associates*, FEA.

$$IVaR(a)=VaR(p+a)-VaR(p)\approx\sum_{i=1}^{n}\frac{\partial VaR}{\partial\omega_i}\Delta\omega_i$$

where $\frac{\partial VaR}{\partial\omega_i}$ give us the marginal changes in VaR associated with marginal changes in the relevant cash-flow elements.

3.2.7.2 Component-VaR (CVaR)

Component VaR (CVaR) investigates the constituent risks that make up a certain total VaR risk. As in the case of IVaR there are three main cases of CVaR:

a. **High CVaR** – where there are high pockets of risk, which contribute strongly to overall portfolio VaR.
b. **Moderate positive CVaR** – where there are moderate pockets of risk.
c. **Negative CVaR** – where there are natural hedges that offset some of the risk of the portfolio.

CVaR is calculated from the following equation:

$$CVaR_i=\omega_i\frac{\partial VaR}{\partial\omega_i}$$

And thus the portfolio VaR is given by:

$$VaR=\sum_{i=1}^{n}CVaR_i$$

In cases where it is more convenient to express a CVaR as a percentage of total VaR, the equation becomes:

$$1=\frac{1}{VaR}\sum_{i=1}^{n}CVaR_i=\sum_{i=1}^{n}\%CVaR_i$$

The primary advantage of using a CVar is its drill-down capability. However, CVaR can be limited by correlation and position-size effects, particularly when positions are significant in size relative to the total portfolio; in which case, the CVaRs will only give an approximate estimate of the effects of the positions concerned on the portfolio VaR.

CHAPTER 4: LIQUIDITY RISK

4.1 Definitions and Concepts[64]

4.1.1 Definitions of Liquidity

a. **At company level** - liquidity refers to solvency.

b. **At market level** - liquidity refers to the ability to trade with little or no cost, risk or inconvenience.

c. **Marketability** – refers to the 'ease of trading an asset' where the easier it is to trade, the lower the liquidity risk of the asset.

d. **Corporate liquidity** – refers to asset and liability liquidity.

e. **Liquidity risk** - the potential loss due to time-varying liquidity costs.

f. **Liability liquidity risk –** the risk arising from funding whereby liabilities cannot be met when they fall due.

g. **Asset liquidity risk** – the risk that an asset cannot be sold due to lack of liquidity in the market.

4.1.2 Defining Market Liquidity

a. **Market liquidity cost** – the cost of trading an asset relative to fair value.

b. **Fair value** - the middle of the bid-ask-spread (the mid-price).

c. **Liquidity cost equation:**

$$L_t(q) = T(q) + PI_t(q) + D_t(q)$$

Where:

$T(q)$	are the known direct trading costs, such as exchange fees, brokerage commissions, and transaction taxes
$PI_t(q)$	is the difference between the transaction price and the mid-price
$D_t(q)$	are the costs of delay if a position cannot be traded immediately

[64] Stange, S. and Kaserer, C. (2009), "Market Liquidity Risk - An Overview," *CEFS Working Paper Series 2009 No. 4*, available at SSRN: http://ssrn.com/abstract=1362537

d. **Liquidity cost dimensions:**

- **Tightness** – is the cost of turning a position around in a short time as measured by the sum of direct trading costs *T* and price impact costs *PI*.
- **Depth** – is the size of the change in an order flow required to change prices by a certain amount.
- **Resiliency** – is the speed with which prices recover from a random shock (the mean reversion speed of liquidity cost).
- **Immediacy** - is the time between submission and settlement of an order and corresponds to the delay cost *D*.

4.2 Measurements of Liquidity Risk[65]

The choice of model used to assess liquidity risk is chosen on the basis of three criteria: the degree of market liquidity, the typical size of a position, and the availability of suitable data.

Nature of Asset	Appropriate Model
Market Liquidity Criteria	
Highly liquid – continuously traded	Liquidity cost models
Moderately liquid – traded with large interruptions	Models incorporating execution delay
Illiquid	Internal models
Size of Position Criteria	
Small position size relative to volumes	Models that ignore the price impact of position size
Medium and large size relative to volumes	Models become increasingly imprecise
Data Availability	
Spread data	Price impact is generally neglected
Transaction data	Price impact approximations are possible
Limit order book data	Price impact can be quite precisely estimated

65 Ernst, C., Stange, S. and Kaserer, C. (2009), "Measuring Market Liquidity Risk – Which Model Works Best?" Center for Entrepreneurial and Financial Studies, Technische Universitat Munchen, *Working Paper 2009 No. 1*; and
Stange, S. and Kaserer, C. (2009), "Market Liquidity Risk - An Overview," *CEFS Working Paper Series 2009 No. 4*, available at SSRN: http://ssrn.com/abstract=1362537

4.2.1 Bid-Ask-Spread Based Models

4.2.1.1 Liquidity Adjusted VaR

The liquidity adjusted VaR-measure was developed by Bangia, Diebold, Schuermann and Stroughair (1998, 1999).[66] This approach measures liquidity risk in terms of the bid-ask-spread and its volatility. In order to determine liquidity risk (deemed to be the worst transaction price); the worst bid-ask-spread is added to the worst mid-price as follows:

$$L - VaR = 1 - e^{(z\sigma_r)} + \frac{1}{2}(\mu_S + \hat{z}_S\sigma_S)$$

Where:

z is the percentile of the normal distribution for the given confidence (commonly 1.645 at 95% confidence)

σ_r is the variance of the mid-price return

μ_S is the mean of the bid-ask-spread

σ_S is the variance of the bid-ask-spread

$\hat{z}_S\sigma_S$ is the percentile of the spread distribution, which accounts for non-normality in spreads.

The advantages of the Bangia *et al.* approach are:

- It is simple to implement as the spread data required is readily obtainable across a range of frequencies;
- It is quick to implement because the liquidity-adjustment is simply added to the existing price risk measures; and
- It is the model of choice when data is scarce.

However, the disadvantages of the approach are:

- It neglects price impacts;
- Liquidity risk may be underestimated for large positions;

[66] Bangia, A., Diebold, F.X., Schuermann T. and Stroughair J.D. (1998), "Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management," Working Paper, Financial Institutions Center at The Wharton School;
Bangia, A., Diebold, F.X., Schuermann T. and Stroughair J.D. (1999), "Liquidity on the Outside," *Risk*, Vol. 12, Pgs. 68-73.

- Liquidity risks may be over-estimated because the approach assumes perfect tail correlation between the spread and prices (assumes that worst liquidity costs and lowest prices occur simultaneously);
- Accounting for non-normality is difficult and thus the approach tends to underestimate liquidity risk.

4.2.1.2 Non-Normal Liquidity Adjusted VaR

Ernst, Stange and Kaserer (2008)[67] developed the non-normal liquidity adjusted VaR approach, which uses non-normal distributions for prices and spreads and thus accounts for skewness and kurtosis. This approach adjusts the liquidity adjusted VaR as follows:

$$L - VaR = 1 - e^{(\mu_r + \tilde{z}_r \sigma_r)}\left(1 - \frac{1}{2}(\mu_S + \tilde{z}_S \sigma_S\right)$$

Where:

μ_r is the mean mid-price return

σ_r is the variance of the mid-price return

μ_S is the mean of the bid-ask-spread

σ_S is the variance of the bid-ask-spread

$\tilde{z}$ is the non-normal-distribution percentile adjusted for skewness and kurtosis according to the Cornish-Fisher expansion as follows:

$$\tilde{z} = z + \frac{1}{6}(z^2 - 1)\gamma + \frac{1}{24}(z^3 - 3z)\kappa - \frac{1}{36}(2z^3 - 5z)\gamma^2$$

where z is the percentile of the normal distribution, γ is the skewness, and κ is the excess-kurtosis.

The advantage of this approach is that it is more accurate than the specification of Bangia *et al.* (1999). However, there are short-comings:

- It generally assumes that positions can be traded at the bid-ask-spread; and
- It assumes that there is perfect correlation between the mid-price return and liquidity costs.

[67] Ernst, C., Stange, S. and Kaserer, C. (2008), "Accounting for Non-Normality in Liquidity Risk," *CEFS Working Paper 2008 No. 14*, available at http://ssrn.com/abstract=1316769

4.2.2 Transactions or Volume Based Models

Liquidity adjusted VaR and non-normal liquidity adjusted VaR models do not consider the impact that trading has on the market price of an asset. Hence the approach of Berkowitz (2000b) takes account of the liquidity effects of historic trades while Cosandey (2001) takes account of the effects of the volume of trades. The basic principle underlying these approaches is that the more responsive the market price is to the trade, the bigger the loss. Although these models are relatively simple to implement, theyare limited in that they are narrow in focus, ignore bid–ask spreads, and ignore transactions costs.[68]

4.2.2.1 Transaction Regression Model

The approach of Berkowitz (2000a,b)[69] assesses the liquidity price impact using a regression of past trades while controlling for other factors as follows:

$$P_{mid,t+1} - P_{mid,t} = C + \theta N_t + x_{t+1} + \varepsilon_t$$

Where:

C is a constant

N_t is the number of shares traded

θ is the regression coefficient (the absolute liquidity cost per share traded)

x_{t+1} is the impact on the mid-price of risk factor changes

ε_t is the error term of the regression

The advantages of the Berkowitz-approach are:

- It integrates the price impact of order size beyond the bid-ask-spread;
- It only uses transaction data for the liquidity measurement;

However, the approach also has the following limitations:

- Calculating the price impact cost from single trades accurately requires intraday data;
- The liquidity measure tends to be relatively imprecise;
- It may underestimate liquidity risk impacts because the approach assumes that price impacts are linear and non-time-varying (which is not the case in actuality);

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68 Dowd, K. (2002), *An Introduction to Market Risk Measurement*, Pg. 132, John Wiley & Sons, UK.

69 Berkowitz, J. (2000a), "Breaking the Silence," *Risk*, Vol. 13, No. 10, Pgs. 105-108;
Berkowitz, J. (2000b), "Incorporating Liquidity Risk Into Value-at-Risk Models," *Working Paper*, University of California, Irvine.

- Further under-estimation is possible because of the assumption of zero liquidity-return correlation (because empirically, positive correlations can be observed); and
- The approach is not suitable in a crisis situation.

Thus although the approach of Berkowitz (2000a,b) provides a method for integrating price impacts of order size in a risk framework, liquidity measurement is approximate.

4.2.2.2 Volume-Based Price Impact

The price impact arising from the volume of trades can be determined from the approach of Cosandey (2001).[70] In this approach, the total value traded in the market is assumed to be constant and apportioned over the number of shares traded (N_t). Thus if a portion of shares are liquidated $n = q / P_{mid,t}$ then the total value traded in the market will then be split over $N_t + n$. Consequently, the net return will be:

$$rnet_{t+1}(q) = \ln\left(r_{t+1} \times \frac{N_t}{N_t + n} \right)$$

In terms of the approach of Berkowitz (2000a):

$$L - VaR(q) = 1 - e^{(\mu_{rnet(q)} + \hat{z}\sigma_{rnet(q)})}$$

The approach of Cosandey has the following advantages:

- It is an improvement over Bangia *et al.* (1999) because it takes account of the price impacts of order size;
- It is computationally simple to undertake and has few data limitations since it makes use of volume data;
- The linear implementation is simple and straight forward.

However, the approach has the following short-comings:

- It requires data on the overall market volume;
- The linearity of the price impact may result in imprecision as empirically, price impacts have been found to be concave not linear. Thus the linear functional form may overestimate liquidity risk for large order sizes.
- It assumes that the amount of trading in the market (N) does not vary over time;

[70] Cosandey, D. (2001), "Adjusting Value at Risk for Market Liquidity," *Risk*, Pgs. 115-118.

- It assumes that liquidity between shares is constant (apart from differences in trading volume); and
- The approach may under-estimate liquidity risk during a crisis.

4.2.3 Weighted Spread Based Models

Weight spread based models use limit order book data to account for the rising liquidity costs that accompany increasing order size. Ernst *et al.* (2009) measure liquidity cost as the "weighted spread," which is the difference between the liquidity costs and the fair market price after liquidating an asset with quantity (*q*) against the limit order book (a "limit order" is an order that can only be executed at, or in excess of, a specified price). Hence, the weighted spread (*WS*) is calculated as follows:

$$WS_t(q) = \frac{a_t(v) - b_t(v)}{P_{mid,t}}$$

where $a_t(v)$ is the volume-weighted ask price of trading *v* shares ($a_t(v)$ is calculated as $a_t(v) = \sum_i \frac{a_{i,t} v_{i,t}}{v}$ where $a_{i,t}$ is the ask-price and $v_{i,t}$ v is the ask-volume of individual limit orders).

4.2.3.1 Limit Order Model

The relative, liquidity adjusted total risk model was proposed by Francois-Heude and Van Wynendaele (2001)[71] and calculated as follows:

$$L - VaR(q) = 1 - e^{(-z\sigma_r)}\left(1 - \frac{\mu(q)_{WS}}{2}\right) + \frac{1}{2}(WS_t(q) - \mu(q)_{WS})$$

Where:

Z	is the normal percentile
σ_r	is the standard deviation of the mid-price return distribution
$\mu(q)_{WS}$	is the average spread for a share with order quantity *q*
$WS_t(q)$	is the weighted spread at time *t*

This approach has the following limitations:

[71] Francois-Heude, A. and Van Wynendaele, P. (2001), "Integrating Liquidity Risk in a Parametric Intraday VaR Framework," *Working Paper.*

- It typically requires intraday data to estimate the price impact function and thus restricts the risk estimation at intraday frequencies;
- Suitable intraday data may not be readily available; and
- Precision is restricted by the size of the order size due to extrapolation.

4.2.3.2 T-Distributed Net Return Model with Weighted Spread

Giot and Grammig (2005)[72] use the weighted spread to derive the T-distributed net return model as follows:

$$rnet_t(q) = r_t \times \left(1 - \frac{WS_t(q)}{2}\right)$$

Thereafter, it is possible to calculate the relative, liquidity-adjusted total risk as follows:

$$L - VaR(q) = 1 - e^{(\mu_{rnet(q)} + z_{ST}\sigma_{rnet(q)})}$$

Where z_{ST} is the chosen percentile of the student *t*-distribution.

The approach of Giot and Grammig has the following advantages:

- The use of weighted spreads means that the price impacts of positions size can be accurately modelled;
- The weighted spread is a precise liquidity measure in a large range of situations;
- It is accurate in markets where asset positions are generally continuously traded;
- It takes account of time variation and non-normality; and
- The correlation between return and liquidity cost does not have to be explicitly modelled because the approach models net-return rather than separating mid-price return and liquidity cost.

However, this approach has the following limitations:

- It requires a transparent limit order book market; and
- It is computationally intensive due to the large amount of data required if weighted spread data must be manually calculated from the full intraday limit order book.

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[72] Giot, P. and Grammig, J. (2005), "How Large is Liquidity Risk in an Automated Auction Market?" *Empirical Economics*, Vol. 30, No. 4, Pgs. 867-887.

4.2.3.3 *Empirical Net-Return Model with Weighted Spread*

In order to circumvent the assumptions associated with t-distributed net returns, Stange and Kaserer (2008c)[73] calculate liquidity risk with the weighted spread based on empirical percentiles instead. Thus the equation becomes:

$$L - VaR(q) = 1 - e^{(\mu_{rnet(q)} + \hat{z}(q) \times \sigma_{rnet(q)})}$$

where $\hat{z}$ is the empirical percentile of the net return distribution.

4.2.3.4 *Modified Risk Models with Weighted Spread*

The non-normal liquidity adjusted VaR approach of Ernst *et al.* (2008) can also be combined with the weighted spread approaches using the Cornish-Fisher equation to derive modified add-on models and modified net-return models.

a. **Modified add-on model with weighted spread** - this modification assumes perfect correlation between mid-price returns and liquidity but takes account for liquidity risk as an add-on per the following:

$$L - VaR(q) = 1 - e^{(\mu_r + \tilde{z}_r \sigma_r)} \left(1 - \frac{1}{2}(\mu_{WS(q)} + \tilde{z}_{WS(q)} \sigma_{WS(q)}\right)$$

   where $\tilde{z}$ is the percentile as estimated with the Cornish-Fisher approximation.

b. **Modified net-return model with weighted spread** - this modification does not rely on the assumption of t-distributed net returns or of a perfect return-liquidity correlation. Hence, the net return is calculated as:

$$L - VaR(q) = 1 - e^{(\mu_{rnet(q)} + \tilde{z}(q) \times \sigma_{rnet(q)})}$$

73 Stange, S. and Kaserer, C. (2008c), "Why and How to Integrate Liquidity Risk into a VaR-Framework," *CEFS Working Paper 2008 No. 10*, available at http://ssrn.com/abstract=1292289

4.3 Assessing Liquidity Risk Models

Ernst, C., Stange, S. and Kaserer, C. (2008) assess the performance of a wide selection of liquidity risk models using comparative back-tests of daily risk forecasts. Their results found:

- The accuracy of risk forecasts is mainly driven by the availability of suitable data.
- Models based on limit order data tend to outperform models based on bid-ask spread or volume data.
- The approaches of Cosandey (2001) and Berkowitz (2000a) should only be used if no other data is available.
- The approaches of Stange and Kaserer (2008c), Giot and Grammig (2005), and the modified Ernst *et al.* (2008) models all produce satisfactory results when limit order book data is available.
- Of these three approaches, the modified Ernst *et al.* (2008) approach performs slightly better than the approaches of Stange and Kaserer (2008c), Giot and Grammig (2005).
- In a case where only bid-ask-spread data is available, the modified add-on model with bid-ask spreads by Ernst *et al.* (2008) is most applicable.

Table overleaf sourced from Stange and Kaserer, (2009) Pg. 24

Model	Short description	Relevant assumptions	Strengths	Weaknesses
Bangia et al. (1999)	• L. measured with bid-ask-spread • Parametric worst liquidity cost added to price risk • Non-normality accounted for through empirical percentiles	• Position size without significant influence • Liquidity cost and price perfectly correlated	• Only spread data required • Simple add-on to existing risk measures	• Underestimation for larger order sizes • Overestimation because correlation less than perfect and spread calculated based on current price • Regime switching (multi-modality) of spread is neglected
Ernst, Stange, Kaserer (2009)	• Improvements on Bangia et al. (1999) • Risk modeled based on net return • Non-normality accounted for by Cornish-Fisher approximation	• Position size without significant influence	• Only spread data required	• Underestimation for larger order sizes • Regime switching (multi-modality) of spread is neglected
Cosandey (2001)	• L. measured as position size relative to traded shares • L. adjustment from worst market price	• No time variation of l. • Price impact linear to relative traded shares • Further l. differences neglected	• Market volume data required and available at all frequencies • Accounts for price impact of order size	• Price impact approximation • Underestimation because deterioration of liquidity in crises neglected
Berkowitz (2000) Jarrow and Protter (2005)	• L. measured from transaction prices, Berkowitz controlling for other risk factor changes	• Cost precisely extractable from transaction data • Linear price impact • Time variation and correlation issues solved by Jarrow, Protter (2005)	• Accounts for price impact of order size	• Intraday data on single transaction prices and volumes required • Price impact only very approximate
Francois-Heude and Van Wynendaele (2001)	• Quantity adjusted L. measured from best five limit-orders • Current market avg. liquidity costs added to price risk with ad-hoc add-on of difference between market and individual liquidity	• No time variation of l. • Liquidity cost and price perfectly correlated	• Accounts for price impact of order size • More precise price impact measurement than when extracted from transaction prices	• Intraday data of best limit orders required • Price impact approximation most precise for small sizes only • Underestimation because deterioration of liquidity in crises neglected • Somewhat arbitrary spread-adjustment leads to overestimation
Angelidis and Benos (2006)	• L. measured by estimating a model of structural liquidity determinants • Worst liquidity cost added to price risk	• Specific structural model • Volume increase in crises • Liquidity cost and price perfectly correlated	• Partially accounts for price impact of order size if larger than worst market volume	• Intraday data required • Assumptions empirically not verified • Overestimation because correlation less than perfect • Complex, time-consuming estimation of parameters
Giot and Gramming (2005) Stange and Kaserer (2008)	• L. measured by weighted spread in limit order book for a specific position size • Risk modeled based on net return	• Worst case perspective (because immediate liquidation) OR Efficient market for liquidity	• Accounts for price impact of order size • Precise liquidity measure • Correlation correctly accounted for	• Only applicable in limit order book markets • If weighted spread data not provided by exchange, intraday data of full limit order book required • Possible overestimation if instant liquidation highly suboptimal

CHAPTER 5: CREDIT RISK

Recommended Reading: Textbook Chapters 8 and 12

5.1 Definitions and Concepts[74]

a. **Credit risk** – the risk of a financial loss arising from a counter-party default.

b. **Sovereign risk** - the risk of a government default.

c. **Counterparty/default risk** - the risk that a counter-party will not pay out on a financial instrument or contract.

d. **Credit event** – a default that may contribute to the degree of credit loss in credit risk models. There are four basic types of credit events:
   - A change in loss rates for a given default (LGD)
   - A change in creditworthiness;
   - A change in the applicable credit spread; and
   - A change in exposure to a particular credit facility.

5.2 Credit Derivatives

A **credit derivative** is a financial instrument whose value is derived from the creditworthiness on an underlying asset. The credit derivative is used to provide insurance against a default by a **reference entity**. The two common categories of credit derivatives are:

a. **Unfunded credit derivatives** - where credit protection is bought and sold between bilateral counterparties. Unfunded credit derivative products include the following:
   - Credit default swap (CDS)
   - Total return swap (TRS)

b. Funded credit derivatives – where the credit derivative is entered into by a financial institution and payments under the credit derivative are funded using some form of securitization such that a debt obligation is issued by the financial institution to support these obligations. Funded credit derivative transactions are often rated by rating agencies. Funded credit derivative products include the:
   - Collateralised Debt Obligation (CDO)
   - Mortgage backed security (MBS)

[74] Basle Committee on Banking Supervision, *Credit Risk Modelling: Current Practices and Applications*, Basle, 1999.

5.2.1 Unfunded Credit Derivative Products[75]

5.2.1.1 Credit Default Swap (CDS)

A **credit default swap (CDS)** is a bilateral contract between a *protection buyer* and a *protection seller* that provides insurance against the risk of default by a third party *reference entity*. The credit default swap relates to the specified debt obligations of the reference entity. The protection buyer obtains the right to sell a bond issued by the reference entity for its par value if a credit event occurs. The bond is referred to as the *reference obligation* and the total par value of the bond is the swap's *notional principle*. Common credit events include:

- Bankruptcy of the reference entity;
- Failure by the reference entity to pay an obligation;
- Default by the reference entity on an obligation;
- Restructure of a covered obligation by the reference entity.

The buyer of the CDS makes periodic payments to the seller for the duration of the life of the CDS or until the occurrence of a credit event. If a credit event occurs then the buyer is entitled to a final accrual payment and the swap is then settled. CDS can be settled either in physical delivery or in cash.

a. **In the case of physical settlement** - the protection buyer delivers the bonds to the seller in exchange for their par value.
b. **In the case of a cash settlement** – the relevant obligation of the reference entity is valued based on the mid-market price (Z) and the cash settlement is then (100-Z)% of the notional principle.

The differences between a CDS and a standard insurance product are:[76]

- The protection buyer does not need to own an underlying obligation of the reference entity.
- The protection buyer does not need to suffer a loss.
- The protection seller has no inherent recourse to the reference entity in the event of default because the reference entity is not a party to the CDS agreement.

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[75] Hull, J.C. (2003), *Options, Futures and Other Derivatives*, 5th ed., Prentice Hall, Chapter 27.
[76] *Documenting Credit Default Swaps on Asset Backed Securities*, Edmund Parker and Jamila Piracci, Mayer Brown, available at http://www.mayerbrown.com/london/article.asp?id=3517&nid=1575.

The present value of the CDS payments is calculated as follows:

$$w\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i + w\pi u(T)$$

Where:

T	is the life of the CDS in years
p_i	is the risk-neutral probability of default at time t_i
$u(t)$	is the present value of annual payments
$e(t)$	is the present value of a payment at time t equal to $t - t^*$ (where t^* is the payment date immediately preceding time t)
$v(t)$	is the present value of the notional principle received at time t
w	is the payments per year made by a CDS buyer
π	is the risk-neutral probability of no credit event during the life of the CDS, measured as $\pi = 1 - \sum_{i=1}^{n} p_i$

If a credit event occurs at time t, then the risk-neutral payoff from the CDS is:

$$1-[1+A(t_i)]\hat{R} = 1-\hat{R}-A(t_i)\hat{R}$$

Where:

$A(t)$	is the accrued interest on the reference obligation at time t as a percentage of the face value
$\hat{R}$	is the expected recovery rate on the reference obligation in a risk-neutral world
$[1+A(t_i)\hat{R}]$	is the risk-neutral expected value of the reference obligation

Thus the present value of the CDS to the buyer is:

$$\sum_{i=1}^{n}[1-\hat{R}-A(t_i)\hat{R}]p_i v(t_i) - w\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i - w\pi u(T)$$

where $\sum_{i=1}^{n}[1-\hat{R}-A(t_i)\hat{R}]p_i v(t_i)$ is the present value of the expected payoff from the CDS.

Hence, the CDS spread (s) is defined as the value of the payments made by a CDS buyer (w) that will make the following equation zero:

$$s=\frac{\sum_{i=1}^{n}[1-\hat{R}-a(t_i)\hat{R}]p_i v(t_i)}{\sum_{i=1}^{n}[u(t_i)+e(t_i)]p_i+\pi u(T)}$$

5.2.1.2 Total Return Swap (TRS)

A total return swap is a contract between two counterparties to exchange the total return (interest payments plus any capital gains or losses for the payment period) on a reference asset for LIBOR plus a spread. The main difference between a TRS and a CDS is that the CDS swap provides protection against specific credit events while the TRS provides protection against the loss of value irrespective of the cause.

5.2.2 Funded Credit Derivative Products

5.2.2.1 Collateralized Debt Obligations (CDO)

A **collateralized debt obligation (CDO)** is a form of credit derivative offering exposure to a large number of entities in a single instrument. Thus CDOs are a method for packaging credit risk. The underlying credit risks are bonds or loans held by the issuer. The security consists of a group of *tranches*. The creator of the CDO normally retains the first tranche and then sells the other tranches in the market. Thus a CDO facilitates the creation of high-quality debt from lower-quality debt. Other forms of CDOs are:

a. **Synthetic CDOs (CSO)** – where the exposure to each underlying company is a credit default swap.
b. **CDOs-squared** - where each underlying credit risk is itself a CDO tranche.

5.2.2.2 Mortgage Backed Securities (MBS)

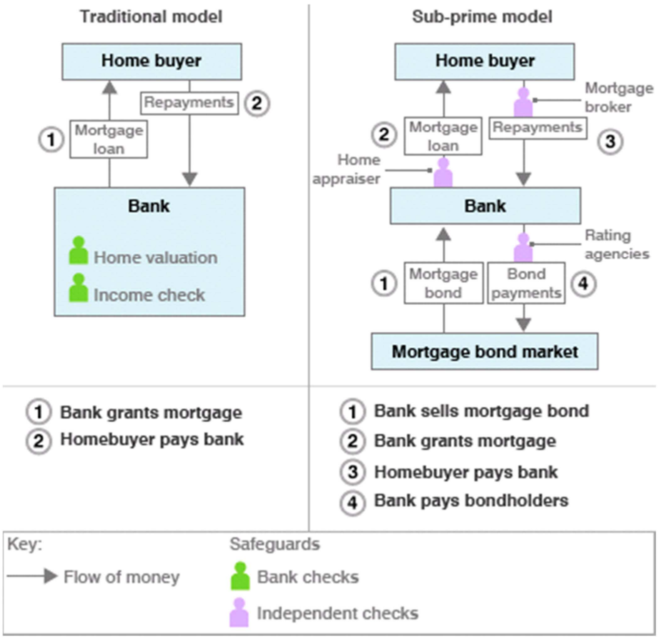
A **mortgage-backed security** is created when a financial institution sells a portfolio of mortgages to investors. The investors in the portfolio purchase units called *mortgage-backed securities*. These units can also be sold on to other investors in a secondary market. An investor owning X% of the portfolio is entitled to X% of the principle and interest cash flows that are received from the mortgages in the portfolio. The mortgages are typically guaranteed by government agencies and thus investors are protected from default. Other variants of MBS' are:

a. **Collateralized mortgage obligations (CMO)** – where the investors are divided into classes, each governed by different principle repayment rules. The objective of a CMO is to create classes that are more attractive to institutional investors than provided by a normal MBS.
b. **Stripped MBS** – where principle payments are separated from the interest payments. All of the principle payments are channelled into principle only securities (*PO*s) and all of the interest payments are channelled into interest only securities (*IO*s). As prepayments increase, the PO becomes more valuable and the IO becomes less valuable (the opposite is the case when the prepayments decrease). Stripped MBS are considered to be risky investments.

MBS are valued by the determination of the *prepayment function,* which describes the expected prepayments on the underlying portfolio of mortgages in terms of the yield curve. A prepayment function of an individual mortgage is unreliable; however, the combination of the functions in a portfolio of mortgages renders the function more predictable. Consequently, valuing MBS is usually undertaken using Monte Carlo simulation.

5.2.2.3 Effect on the Banking Sector

The explosion of MBS resulted in the development of the *Originate-to-Distribute* banking model. In the traditional banking model (*Originate-to-Hold*), banks would provide a mortgage loan in exchange for a series of repayments. Hence the bank accepted the risk of default in the case of non-payment. However, in the *Originate-to-Distribute* banking model, the bank repackages and sells the rights to the mortgage payments on to investors in the market. Hence, the credit risk is passed on to financial markets instead, creating moral hazard incentives because the bank can derive the financial benefits from the origination fees without bearing the credit risk of the borrowers.



Source: http://en.wikipedia.org/wiki/Subprime_mortgage_crisis

Purnanandam (2010)[77] shows that banks that were highly involved in the *Originate-to-Distribute market* prior to the sub-prime mortgage crisis tended to originate very poor quality mortgages. The degree of undisciplined lending was higher among poorly capitalized banks, banks that relied less on demand deposits, and capital-constrained banks. Thus, the empirical evidence indicates that the lack of pre-mortgage screening incentives in combination with moral hazard incentives significantly contributed to the sub-prime mortgage crisis.

5.2.3 Credit Risk Models[78]

Most credit risk models can be defined as either structural models or reduced-form models:[79]

a. **Structural models** - use an entity's structural variables (such as asset and debt values) to determine the time of default by describing the evolution of the zero curve over time.

b. **Reduced form models** - do not consider the relation between default and firm value explicitly. In contrast to structural models, the time of default is determined by the first jump of an exogenous process inferred from market data.

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[77] Purnanandam, A. (2010), "Originate-to-Distribute Model and the Subprime Mortgage Crisis," *AFA 2010 Atlanta Meetings Paper*, available at SSRN: http://ssrn.com/abstract=1167786

[78] Arora, N., Bohn, J.R. and Zhu, F. (2005), "Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models," *Journal of Investment Management*, Vol. 3, No. 4, available at SSRN: http://ssrn.com/abstract=723041

[79] Elizalde, A. (2005), "Credit Risk Models II: Structural Models," CEMFI, Spain, available at www.abelelizalde.com/pdf/survey2%20-%20structural.pdf

Hence the principle differences between structural models and reduced-form models are:

- Structural models treat defaults as endogenous but reduced form models treat defaults as exogenous.
- In structural models, the value of an entities assets and liabilities at default determine recovery rates but in reduced form models, recovery rates are specified exogenously.
- Reduced form models assume that an entities default time is unpredictable but can be modelled using a sudden jump in default intensity, which is a function of latent state variables.

5.2.3.1 Merton Model

The **Merton model**[80] is considered to be the first structural model of credit risk. In this model, an entities' asset value is modelled as a lognormal process based on the assumption that the entity will default if the asset value (*A*) falls below a specified default boundary (*X*). The default is allowed at a point in time (*T*). The equity (*E*) of the entity is then modelled as a call option on the underlying assets where the value of the equity is calculated as follows:

Assets = equity + risky debt

$$A(t) = E(t) + D(t)$$

Hence the value of the equity is computed using the Black-Scholes-Merton formula for the value of a call:

$$E = AN(d_1) - Xe^{-rT}N(d_2) \qquad \text{assuming } t=0$$

where:

$$d_1 = \frac{ln\left(\frac{A}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

A is the initial asset value of the entity

X is the default barrier for the entity (if the entities' asset value *A* is below *X* at the terminal date *T*, then the entity is in default)

r is the risk-free rate

σ is the volatility of the asset returns

N(d) is the cumulative probability of the standard normal density function below *d*

80 Merton, R. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, Vol. 29, Pgs. 449-70.

The formula for the "delta" of the put option is $N(d_1) - 1$ and $N(-d_2)$ is the risk-neutral probability of default. Hence the value of the debt (*D*) is $D = A - E$ and the value of the spread (*s*) is given by:

$$s = -\frac{1}{T} ln\left[N(d_2) + \frac{A}{X} e^{rT} N(-d_1)\right]$$

Worked Example:[81]

Calculate the value of a credit spread where the initial asset value of the entity (*A*)=R100, σ=40%, the default barrier for the entity (*X*)=75, *r*=5%, *T*=one year, the yield to maturity on the risky debt is 10.34%, and the credit spread is 5.34%.

Thus the risk-adjusted probability of default is 26% calculated from:

$d_1 = 1.0442$

$d_2 = 0.6442$

$N(d_1) = 85\%$

$N(d_2) = 74\%$

$N(-d_1) = 1 - N(d_1) = 15\%$

Hence, the risk neutral or risk adjusted default probability is $N(-d_2) = 1 - N(d_2) = 26\%$. The equity value is R32.37 based on $E = 100N(d_1) - 75e^{-0.05(1)}N(d_2) = 32.367$ and the value of the risky debt is R67.63 based on $D = A - E = 100 - 32.367 = 67.633$.

Therefore, the value of the credit spread is 5.34%:

$$s = -\frac{1}{1} ln\left[N(d_2) + \frac{100}{75} e^{0.05(1)} N(-d_1)\right] = 5.34\%$$

5.2.3.2 Vasicek-Kealhofer (VK) Model

The **Vasicek-Kealhofer (VK) model**[82] is a structural model that treats equity as an option on the underlying assets of an entity. The *Distance-to-Default term-structure* (*DD*) is determined empirically using the market asset value, asset volatility, and the *default point term-structure* (the default barrier at different points in time). The term-structure is then translated to a physical default probability by

[81] Example sourced from Gray, D. and Malone, S.W. (2008), *Macrofinancial Risk Analysis*, John Wiley & Sons, UK, Pgs. 65-66.

[82] Kealhofer, S. (2003a), "Quantifying Credit Risk I: Default Prediction," *Financial Analysts Journal*, January/February; Kealhofer, S. (2003b), "Quantifying Credit Risk II: Debt Valuation," *Financial Analysts Journal*, March/April; Vasicek, O. (1984), "Credit Valuation," *White Paper*, Moody's KMV.

mapping the difference between the *distance-to-default term-structure* and historical defaults based on the following equation:

$$DD_T = \frac{\log\left[\frac{A}{X_t}\right] + \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

Where X_t is the default-probability term-structure. The default probability generated by the VK model is called an *Expected Default Frequency* (*EDF*) credit measure. Once the *EDF* term-structure has been derived, a *Cumulative EDF term structure* can be calculated up to any term (*T*) referred to as $CEDF_T$. This can then be converted into a risk-neutral cumulative default probability ($CQDF_T$) using the following equation:

$$CQDF_T = \phi\left[\phi^{-1}[CEDF_T] + \lambda\sqrt{R^2}\sqrt{T}\right]$$

Where:

R^2 is the square of correlation between the underlying asset returns and the market index returns

λ is the market Sharpe ratio

The spread (*s*) of a zero-coupon bond is obtained as:

$$s = -\frac{1}{T}\log\left[1 - LGD \times CQDF_T\right]$$

Where:

LGD represents the loss given default in a risk-neutral framework

$CQDF_T$ is the risk-neutral cumulative default probability

The advantage of the VK model is that it can accommodate five different types of liabilities: short-term liabilities, long-term liabilities, convertible debt, preferred equity, and common equity.

5.2.3.3 Hull-White (HW) Model

The **Hull-White model**[83] is a reduced-form model that provides an exact fit of a risk-neutral default risk density using a cross-section of bonds with various maturities. The HW model thus provides a methodology for valuing credit default swaps in cases where the payoff is contingent on default by a single reference entity and there is no counterparty default risk. Instead of using a risk rate as the default probability, the HW model makes use of a default density, which consists of the unconditional cumulative default probability in a particular period. Through a process of mean-reversion and bootstrapping, the model uses zero-coupon bond prices and zero-coupon risk-free bond prices to recursively generate the default densities. Thereafter, premium of a credit default swap contract can be calculated from the default density term-structure. Thus the CDS spread (s) can be calculated as:

$$s = \frac{\int_0^T \left[1-\hat{R}(1+A(t))\right] q(t)v(t)\Delta t}{\int_0^T q(t)\left[\mu(t)+e(t)\right]\Delta t + \pi u(t)}$$

where:

T is the life of the CDS contract

$\hat{R}$ is the expected recovery rate on the reference obligation in a risk-neutral world

$q(t)$ is the risk-neutral default probability density at time t

$A(t)$ is the accrued interest on the reference obligation at time t as a percent of face value

π is the risk-neutral probability of no credit event over the life of the CDS contract

v is the total payments per year made by the protection buyer

$e(t)$ is the present value of the accrued payment from previous payment date to current date

$u(t)$ is the present value of the payments at time t

The bond data is used to calculate the risk-neutral default probability density (q) as follows:

$$q_j = \frac{G_j - B_j - \sum_{i=1}^{j-1} q_i \alpha_{ij}}{\alpha_{jj}} \text{ where } \alpha_{ij} = v(t_i)\left[F_j(t_i) - R_j(t_i)C_j(t_i)\right]$$

[83] Hull, J., and White, A. (2000), "Valuing Credit Default Swaps: No Counterparty Default Risk," *Working Paper*, University of Toronto.

Where:

α_{ij} is the present value of the loss on a defaultable bond (*j*) relative to an equivalent default-free bond at time t_i

$v(t_i)$ is the present value of a payment at time t_i

F_j is the risk-free value of an equivalent default-free bond at time t_i

R_j is the recovery rate on the claim made on the *j*th bond

C_j is the claim made on the *j*th bond in the event of default at time t_i

5.3 Assessing Liquidity Risk Models

Arora *et al.* (2005) empirically test the success of the Merton model, the Vasicek-Kealhofer (VK) model, and the Hull-White (HW) model. The results show that:

- The VK structural model tends to outperform the HW reduced-form model.
- The HW model outperforms the Merton model when an entity issues a large number of bonds.
- The HW model also outperforms (in terms of explaining the cross-sectional variation of CDS spreads) the more sophisticated VK structural model when an entity issues more than 10 low risk bonds.
- However, the error in terms of the difference between actual and predicted levels of spreads was much larger for the HW model than for the VK model.
- The VK model outperforms the Merton and HW models in terms of default predictive power.
- The performance of the VK model is more consistent across large and small entities.
- The performance of the HW and Merton models worsens across larger entities.

CHAPTER 6: OPERATIONAL RISK

Recommended Reading: Textbook Chapters 10, 14 and 15

6.1 Definitions and Concepts

In recent decades risk management has been dominated by the assessment and hedging of market risk, liquidity risk, and credit risk. However, globalisation, deregulation, and increased financial sophistication coupled with unpredictable shocks arising from fraud, system failures, terrorism etc. have shown that operational risks are just as important.

Basel II thus defines **Operational Risk** as - *the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events.*[84]

Operational risk is a wider category of risk as it includes aspects not covered by market risk, liquidity risk, and credit risk. Thus operational risk may consider personnel, systems, processes, legal, physical, and environmental risks. However, strategic and reputational risks are excluded.

6.2 Basel II Methods of Operational Risk Management[85]

Basel II was agreed in June 2004 and updated in November 2005. The aim of Basel II is to provide common standards of regulation and supervision in order to promote global financial stability. Thus the framework seeks to align the capital requirements to the risks in the banking sector, and to adapt regulations to the complexities of risk management in an increasingly globalised financial environment. Basel II proposes three measurement methodologies for calculating the operational risk capital requirements: the Basic Indicator Approach, the Standardized Approach and the Advanced Measurement Approach.

6.2.1 Basic Indicator Approach

The Basic Indicator Approach is most applicable to small credit institutions that cannot afford the sophisticated monitoring needed for the other approaches. The Basic Indicator Approach requires the credit institution to have capital requirement for operational risk equal to 15% of the three year average sum of net interest income and net non-interest income (called the *Exposure Indicator*) calculated as follows:

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[84] Basel Committee on Banking Supervision (2003), "The New Basel Capital Accord," *Consultative Document.*
[85] *Guidelines on Operational Risk Management*, Oesterreichische Nationalbank in Cooperation with the Financial Market Authority, Thonabauer, G. and Nösslinger, B. (eds.), Chapter 4, 2006.

$$C_{BIA} = \alpha \times EI$$

where:

C_{BIA} is the credit institution's capital requirement

α is the 15% capital factor

EI is the Exposure Indicator

6.2.2 Standardized Approach

The Standardized Approach is a more advanced method compared to the Basic Indicator Approach. In the Standardized Approach a credit institution's activities are subdivided into the following eight standardized business lines:

a. Corporate finance
b. Trading and sales
c. Retail brokerage
d. Commercial banking
e. Retail banking
f. Payment and settlement
g. Agency services
h. Asset management

Each business line is then assigned a relevant indicator (net interest income and net non-interest income) that corresponds to the level of business operations and thus the relevant size of operational risk exposure. The capital requirement for a given business line corresponds to a fixed percentage (the *Beta Factor*) of the relevant exposure indicator (as described under the Basic Indicator Approach). The Beta Factors for each business line are as follows:

<table>
<tr><th>Business Line</th><th>Beta</th><th>Beta Factor</th></tr>
<tr><td>Corporate finance</td><td>β_1</td><td rowspan="3">18%</td></tr>
<tr><td>Trading and sales</td><td>β_2</td></tr>
<tr><td>Payment and Settlement</td><td>β_6</td></tr>
<tr><td>Commercial banking</td><td>β_4</td><td rowspan="2">15%</td></tr>
<tr><td>Agency services</td><td>β_7</td></tr>
<tr><td>Retail broking</td><td>β_3</td><td rowspan="3">12%</td></tr>
<tr><td>Retail banking</td><td>β_5</td></tr>
<tr><td>Asset management</td><td>β_8</td></tr>
</table>

Hence the capital requirement for each business line is then determined by the following formula:

$$C_{STA} = \sum_{i=1}^{8} C_{STA,i} = \sum_{i=1}^{8} \beta_i \times EI_i$$

C_{STA} is the capital requirement of the credit institution under the standardized approach

$\sum_{i=1}^{8} C_{STA,i}$ is the sum of capital requirements in the individual business lines

$\sum_{i=1}^{8} \beta_i \times EI_i$ is the sum of the products of net interest income and net non-interest income (the Exposure Indicator) for the individual business lines and the Beta Factors assigned to them

6.2.3 Advanced Measurement Approaches

The Advanced Measurement Approaches (AMAs) may be applied from January 1, 2008 and are risk-sensitive methods for measuring operational risk using measurement techniques developed by each individual credit institution.

6.2.3.1 Qualitative Measures

In contrast to the Basic Indicator and Standardised Approaches, use of the Advanced Measurement Approach requires the use of additional qualitative standards, which are:

- The credit institution must have an internal operational risk measurement system that is integrated into its day-to-day risk management processes.
- The credit institution must have an independent risk management function for operational risk (whose functions will include regular reporting of operational risk exposures and loss experience) and must have procedures for taking appropriate corrective action.
- The credit institution must have a well documented risk management system, as well as routines in place for ensuring compliance and policies for the treatment of non-compliance.
- The credit institution's operational risk management processes and measurement systems must be subject to regular reviews that are performed by internal and/or external auditors.
- The validation of the operational risk measurement system by the competent authorities will include the following:
    - Verification that the internal validation processes are operating satisfactorily.
    - Ensuring that data flows and processes associated with the risk measurement system are transparent and accessible.

6.2.3.2 Quantitative Measures

In addition to qualitative measures, the Advanced Measurement Approaches also requires the credit institution to observe and comply with quantitative requirements.

a. The following **pre-implementation criteria** are applicable:
    - The credit institution must be able to prove that its selected measurement method is capable of capturing potentially severe tail events.
    - The calculated capital requirement comprises both expected loss (EL) and unexpected loss (UL), unless the credit institution can show that expected loss is adequately captured in internal business practices.
    - The credit institution must prove that the operational risk measure achieves a one year confidence interval of 99.9%.

b. The operational risk measurement system must have comprehensive documentation and support of the following **key elements** in order to meet the required standards:
    - **Internal data:**
        - A minimum historical observation period of five years (except when the credit institution first adopts the AMA, when a three-year data series is acceptable).
        - The credit institution must have documented procedures for assessing the ongoing relevance of historical loss data.

    - The internal loss data must capture all material activities and exposures from all appropriate subsystems and geographic locations (any excluded activities and exposures must be justified).
    - Appropriate minimum loss thresholds for internal loss data collection must be defined.
    - Recording of a loss event must include gross loss amounts, the date of the event, any recoveries of gross loss amounts, and descriptive information about the drivers or causes of the loss event.
    - Losses related to credit risks are to be separately identified in the operational risk databases.
- **External data:**
    - The credit institution must have a systematic process to determine the situations for which external data must be used.
    - The methodologies used to process the external data must be documented.
    - The conditions and practices for using external data must be regularly reviewed (internally and externally) and documented.
- **Scenario analyses:**
    - The credit institution's exposure to high severity loss events must make use of scenario analyses.
    - The scenario analysis must be based on expert opinion in conjunction with external and/or internal data.
    - The scenario analysis will produce assessments on potentially severe losses, which can be expressed as a statistical loss distribution.
    - These assessments must be validated over time and adjusted as needed to better reflect reality.
- **Other factors (Business Environment and Internal Control Factors):**
    - The credit institution's risk assessment system must capture key business environment and internal control factors that can influence its operational risk profile.
    - Thus the credit institution's risk assessment is future-oriented.
    - The key drivers of risk must be justified, based on experience and expert judgment.
    - The sensitivity and weightings of risk estimates must be assessed.
    - Improvements in risk controls, as well as increases in risk due to greater complexity of activities or increased business volume must be captured.
    - This framework must be documented and subject to independent internal and external review.

- These assessments must be validated over time and adjusted as needed to better reflect reality.

6.3 Real Options Valuation[86]

6.3.1 Definitions and Concepts

a. **Real Options Analysis** (**ROA)** or **Real Options Valuation (ROV)** values a company's strategic flexibility using option valuation techniques.

b. A real option can be **defined as:** *the right but not the obligation to take an action at a predetermined cost for a predetermined period of time.*

c. Real options are commonly **used to**:

- Identify investment decisions
- Value strategic decisions
- Prioritize projects
- Optimize investment value
- Time effective execution
- Manage existing or developing strategies

d. Real options are commonly **used by**:

- Airline industries
- Automobile manufacturers
- Computer industries
- Mergers and acquisitions
- Oil and gas industries
- Pharmaceutical industries
- Telecommunications industries
- Real estate companies
- Shipping industries
- Utility companies

e. The **common types** of real options are:

- Option to abandon a project – an American put option
- Option to contract (scale back) a project - an American put option

[86] Copeland, T. and Antikarov, V. (2002), *Real Options Analysis: A Practitioner's Guide*, Thomson Texere, New York.

- Option to delay a project- an American call option
- Option to expand a project - an American call option
- Option to switch projects – a portfolio of American call and put options
- Compound options – options on options
- Compound rainbow options – options driven by many sources of uncertainty

6.3.2 Comparison with DCF and NPV

The most significant differences between real options valuation techniques and traditional net present value (NPV) approaches are:

Traditional Approaches	Real Options Approach
Management has only one choice	Management have multiple choices
Projects have limited outcomes	Projects have multiple outcomes
Decisions are made at the start	Decisions are dynamic
Ignores management flexibility	Assumes that management is active and can modify the project as necessary
Assumes total commitment of resources	Considers management's ability to respond to changing situations
Management are passive - do not alter strategic decisions to meet changing circumstances	Management is active and responds to each outcome
Changing circumstances and risk profiles are accounted for by altering the discount rate or the projected cash flows	Changing circumstances and risk profiles are accounted for by using option pricing techniques to risk-adjust the probabilities

Given these theoretic and applied differences, the real options value of a project is typically higher than the net present value. This is mainly because the real options value includes management's strategic flexibility to maximise benefit and mitigate loss as the project adapts throughout its lifetime.

6.3.3 Valuation Inputs

The five inputs required to conduct a real option valuation are similar to those required to perform a valuation of a financial option:

a. **Value of the underlying risky asset** – the project, investment, or acquisition (an important difference between a real option and a financial option is that a real option cannot affect the value of the underlying asset).

b. **Exercise price** – the amount of money invested to purchase the asset (call option) or the amount of money received if selling the asset (put option).

c. **Time to expiration**

d. **Volatility of the underlying asset**

e. **Risk-free rate**

The impact of changes in these key inputs on a real option can be summarized as follows:

Input	Real Option
Increase in PV of the project	Higher real option value
Increase in investment costs	Lower real option value
Longer time to expire	Increase real option value
Increase in volatility	Increase real option value
Increase in the risk-free rate	Increase real option value

Although real options use the same basic inputs as financial options there are significant differences between the two approaches:[87]

Financial Options	Real Options
Short maturity (usually in months)	Long maturity (usually in years)
Underlying asset is a financial asset	Underlying asset are free cash flows
Option value cannot be controlled by manipulating asset prices	Strategic option value can be increased by management decisions and flexibility
Values are usually small	Values are typically substantial
Competitive and market effects are irrelevant	Competitive and market effects have strategic value
Been traded for over three decades	Developed in the last decade
Usually solved using closed-form partial differential equations and simulation techniques for exotic options	Usually solved using closed-form equations and binomial lattice techniques with simulation of the underlying variables, not on the option analysis
Marketable and tradeable with comparative pricing	Not marketable and proprietary with no comparatives
Management assumptions and actions have no impact on the value	Management assumptions and actions drive the value

[87] Mun, J. (2006), *Real Options Analysis*, 2nd ed., John Wiley & Sons, USA, Pg. 110.

6.3.4 Real Options Process[88]

The valuation of a real option typically consists of the following steps:

a. Qualitative management screening – management produce the initial list of projects or investments that are to be analyzed.
b. Time-series and regression forecasting – time-series analysis or multivariate regression is used to produce a forecast.
c. Base case NPV analysis – a discounted free cash flow (DCF) model is produced to derive a static base-case NPV for each potential project or investment.
d. Monte Carlo simulation – first, sensitivity analysis is applied to the DCF model to uncover the key uncertainty drivers of the NPV; and second, correlated Monte Carlo simulation is conducted to approximate the real-world behavior of these key variables.
e. Real options problem framing – the strategic optionalities are framed.
f. Real options modeling and analysis – the results of the Monte Carlo simulation are used to produce the real options calculations.
g. Portfolio and resource optimization – in the case of multiple projects or investments, stochastic optimization is used to create an optimal portfolio mix.
h. Reporting and update analysis.

6.3.5 Valuation Methods[89]

Real options make use of the same valuation techniques as financial options. However, in a real option setting, these techniques have unique advantages and limitations.

6.3.5.1 Black Scholes and Simulation Models

Closed-form solutions such as Black-Scholes approaches are relatively simple to apply to real options problems but suffer from the following drawbacks:

- They are difficult to explain because they are reliant on stochastic calculus mathematics
- They have limited modelling flexibility
- They are exact for European options but are only approximations for American options
- Many exotic and Bermudan options cannot be solved using this approach
- The calculations are difficult to explain in a management report

6.3.5.2 Binomial Models

Using binomial lattices to value real options have the following advantages:

- In industry, real options are most commonly valued using binomial lattices
- Binomial lattice models are easier to use and implement than Black-Scholes models

[88] Ibid, Chapter 4.
[89] Ibid, Chapter 5.

- Binomial lattices can model any type of option
- Binomial lattices use simple algebra rather than stochastic calculus
- *In the limit* results obtained from a binomial lattice are the same as those derived from closed-form solutions

However, the main draw-backs of using lattices are:

- They can require significant computing power
- They can require the use of many lattice steps to obtain good approximations (100 to 1000 steps)

Worked Example:[90]

In December 2002, a developer was considering purchasing a contractual option to build an office complex on a vacant piece of land. The option would expire in two years – if the developer wishes to build the property, he must do so before the end of 2004. Assume for simplicity that the office complex can be built instantaneously once the developer decides to invest. The construction cost is R95 Million at the end of 2002, but would increase each year at the (continuously compounded) risk-free rate of 5% (i.e. the cost would be R99.87 Million at the end of 2003 or R104.99 at the end of 2004).

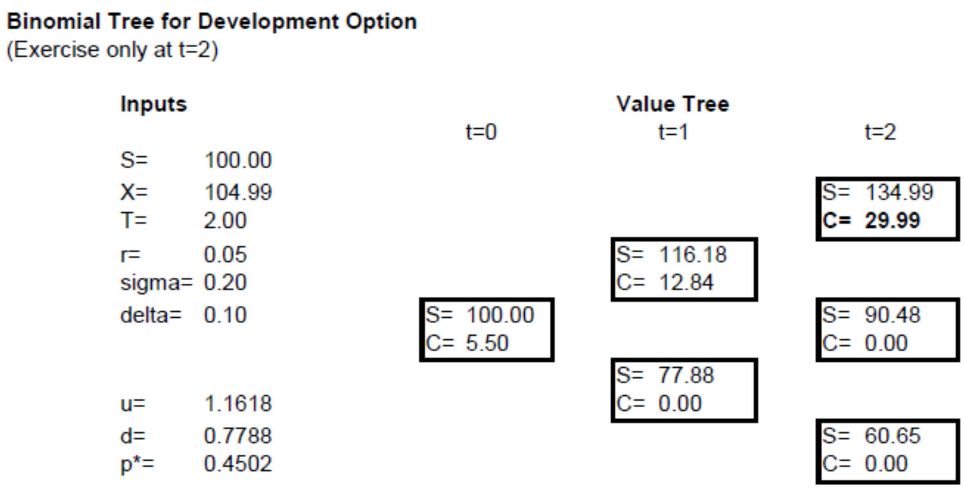
It is expected that a developed property could generate an average after-tax cash flow at a rate of R10 Million per year at the beginning of 2003, but that the expected cash flow decreases at a (continuously compounded) rate of 2% per year. This continuous decline in cash flow generation reflects the effect of economic depreciation of the property as well as the impact of new office construction in the area. Assume an infinite time horizon, and disregard any future managerial actions (such as renovations, expansions, etc.) that may impact the cash flows such that there is only a single decision: to develop the property or not.

In addition, assume that based on market returns for comparable properties in the same geographic region, a (continuously compounded) capitalisation rate of 8% should be used to discount future cash flows. Given the simple structure of this problem, the developed property can be valued as a declining perpetuity. The value at the initial decision point is simply R10 / (.08 + .02) = R100 Million. The value at the end of 2003, assuming the expected case of a 2% decline in the cash flow, would be $R10e^{-0.02/(0.08+0.02)}$ = R98.02, and this value will continue to decline by 2% per year assuming the expected decline in cash flows materialises and the capitalisation rate stays constant (which implies the level of risk, the risk premium, and the risk-free rate all remain constant).

[90] Example sourced from Triantis, A.J. (2003), "Real Options," in Handbook of Modern Finance, Logue, D. and. Seward, J. (eds), New York, Research Institute of America.

In the first stage of the real options analysis, a Binomial Tree is generated. A standard way of setting up a binomial tree is to assume that during each time period of length *T* the value of the underlying asset either increases by a factor of *u* or decreases by a factor of *d*. These return factors are chosen to represent two points that are a standard deviation above or below the "forward value" for the property, i.e. a value that could (at least conceptually) be locked in at the present time for purchasing a developed property in *T* years.Using the parameter values and assuming for simplicity that the two-year horizon of the problem is split into two time periods, each with duration (*T*) of one year, *u*=1.1618 and *d*=0.7788.

Starting with the initial value of the developed property of R100 Million, and multiplying by the return factors *u* and *d*, the two possible values of the developed property at the end of the first year (*t*=1) are 116.18 and 77.88. The tree generation procedure continues by multiplying each of these two values by *u* and *d* in order to obtain the values for the end of the second year (*t*=2). Note that there are only three, rather than four, values (134.99, 90.48, 60.65) at the end of the second year since it is irrelevant whether the underlying asset value goes down first and then up, or vice-versa. In other words, the branches of the tree "recombine," which is an attractive computational feature of such trees. If the two-year horizon of the decision problem were broken down into 24 monthly periods, rather than just two yearly periods, there would be 25 possible values for the underlying asset at the end of the second year, providing a closer approximation to the full distribution of possible values.



Once the values of the underlying asset have been generated, the next step is to calculate the value of the option at the maturity date under each of the value scenarios. This simply requires subtracting the exercise price of 104.99 (i.e. the cost of development) from the developed property value at the end of the two year horizon if the option is *in-the-money* (i.e. when the property value is 134.99, the option value at the maturity date is 29.99), or else setting the option value equal to zero if the option is *out-of-the-money*, i.e. if the property should not be developed.

In the next step of the analysis, an induction process is used to work backwards through the tree to first calculate the option value at the end of the first year (t=1) under each of the two possible value scenarios, and then to calculate the option value at the initial date (t=0). These option values can be calculated using a DCF-type technique, but with a twist. In a conventional DCF calculation, the expected cash flow is estimated, and then discounted at a rate that reflects the risk of the cash flows. In contrast, in the binomial model, the expected value of the option is adjusted to account for the option's risk, and is then discounted at a risk-free rate.

To see how, and why, this is done, start by looking more carefully at the first year of the binomial tree. The value of the developed property at the initial date is R100 Million, which is the present value of all the future expected cash flows obtained from the property. This present value could alternatively be broken down into the present value of the cash flows during the first year, which can be shown to equal 100(1-e-), or R9.52 Million, and the present value of the expected property value at the end of the first year, which is 100e-, or R90.48 Million. The latter implies that the expected property value at t=1 must be (100e-)$e^{0.08}$, or 100$e^{-0.02}$ = R98.02 Million (since the discount rate for the property is 8%). Given the binomial model structure with a value of R116.18 Million in the up state and R77.88 Million in the down state at t=1, the probability (p) of the property value increasing must be approximately 53% (98.2=(0.53)116.18+(1-0.53)77.88).

Now, consider that, while the typical practice in DCF analysis is to account for risk by adjusting the discount rate, there is nothing in financial theory that requires that the denominator, rather than the numerator, incorporate the risk adjustment. Risk can be accounted for in the numerator by decreasing the expected cash flow to reflect a penalty or discount for risk, and the same valuation should be obtained as if the discount rate were risk-adjusted.

A conceptually appealing way of understanding this procedure is that there is a risk-free amount, called the *Certainty Equivalent Value* (CEV), which would be less than the expected value of the cash flow, but that investors would view as equivalent to the risky cash flow. In calculating the present value, the CEV should be discounted at a risk-free rate, since it is indeed risk-free. Thus, risk is accounted for in the numerator rather than the denominator. In the property development example, the CEV at t=1 must be R95.12, since 95.12$e^{-0.05}$ = R90.48, the present value at t=0 of the t=1 property value (note that the risk-free rate of 5% is used for discounting).

The numerator could alternatively be adjusted for risk as follows. *Risk-adjusted probabilities* (often referred to as *risk-neutral probabilities*) could be derived, such that, when they are multiplied by the value of the underlying asset in the up and down nodes at t=1, a value of R95.12 Million (which is equivalent to the CEV) is obtained. The risk-adjusted probability, p^*, of an increase in the property value during the first year must solve the equation p^*=(116.18)+(1-p^*)(77.88)=R95.12. The risk-adjusted probability, p^*=45%, is less than the actual probability of a value increase (p=53%)

calculated earlier. This suggests that the expected cash flow is being penalized or discounted for risk by reducing the probability of a value increase, while increasing the probability of a value drop.

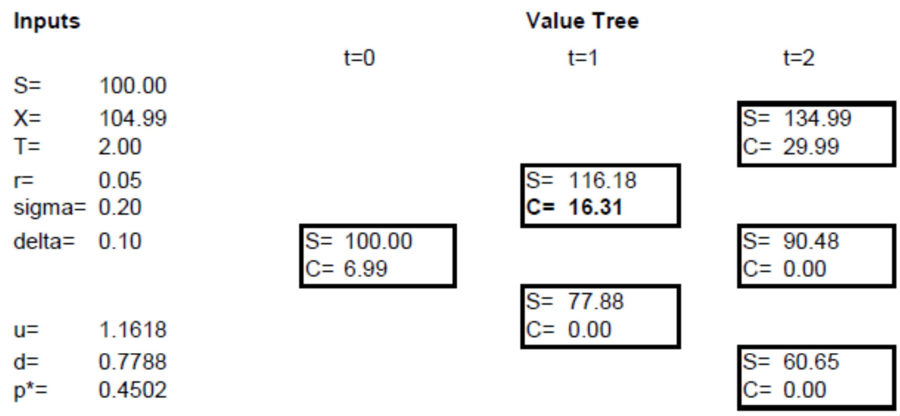
The point of deriving these risk-adjusted probabilities is that they allow for a simple way to value options. As will be shown later, the risk level of options can be very difficult to estimate accurately, and thus risk-adjusting discount rates in a binomial tree can be extremely challenging and prone to error. Instead of following the traditional DCF approach, risk-adjusted probabilities can be used to calculate a risk-adjusted expected value of the option, and then a risk-free rate can be used for discounting.

In the context of the development option, the risk-adjusted probabilities of an increase and decrease in the value of the developed property were found to be 45% and 55%, respectively. While these probabilities were calculated for the first year of the tree, they also apply to the second year of the tree. Starting at t=1, the value of the option when S=R116.18 Million can be calculated as the risk-adjusted expected value of the option at the end of the second year, discounted back to t=1 at the risk-free rate. This option value = $(0.45 * 29.99+0.55*0)e^{-0.05}$ = R12.84 Million). The option value at t=1 when S=R77.88 Million is simply zero since the option values for the corresponding up and down nodes at t=2 are both zero. Going backwards in time to the initial date, the option value is calculated as $(0.45*12.84+0.55*0)e^{-0.05}$ = R5.50 Million.

However, the valuation of the development option has implicitly assumed that the option would not be exercised before the end of the two-year period. Yet, given that a significant cash flow is foregone by waiting to develop, it is quite possible that, under certain circumstances, the option should be exercised before the option's maturity date. Fortunately, this ability to "exercise early" can be easily accommodated within the binomial option valuation model, an important advantage relative to the Black-Scholes formula.

Using the two-period binomial model constructed in the previous section, we must check for "early-exercise" at two points: at t=0 (i.e. immediate development) and at t=1 (develop in one year's time). In the high value scenario at t=1 (S=R116.18 Million), the option value was calculated to be R12.84 Million. However, this option value, which considers the value of the option at the end of the second year and thus incorporates the benefit of waiting for more information, is lower than the profit gained from developing the property at the end of the first year, which is R16.31 Million (R116.18 Million minus the cost of development at t=1, R99.87 Million). The value of the option in this scenario is thus R16.31 Million, reflecting the optimal decision to develop). For the lower value scenario at the end of the first year, the option to develop is out-of-the-money, and thus the developer would hold on to the option for another year (though, in this example, the option has zero value).

Binomial Tree for Development Option with Early Exercise



Stepping back to the initial date (t=0), the property could either be developed immediately or held on to for at least one year. Immediate development has an NPV of R5 Million. The value of the option to defer development is simply the present value of the expected option value at t=1, where the option value when S=R116.18 Million at t=1 now reflects the optimal exercise decision in that scenario. This value of R6.99 Million= $[(0.45*16.31+0)e^{-0.05}]$, is larger than the value of immediate development, and thus the developer should delay development for one year. Note that the option value at t=1 is higher than that computed in the previous section (R5.50 Million). The difference between these two option values represents the value of being able to exercise the option early. In general, early exercise is likely to occur if the option can become *deep-in-the-money* before the maturity date, if the yield on the underlying asset is large, and if the volatility is relatively low. When these conditions are present – and they often are in real options problems - using the Black-Scholes formula will understate the true value of the option.

CHAPTER 7: FAILURE OF FINANCIAL RISK MANAGEMENT: THE CREDIT CRISIS OF 2007-2008

The Financial Crisis Inquiry Commission was created to "examine the causes, domestic and global, of the current financial and economic crisis in the United States." The Commission was established as part of the Fraud Enforcement and Recovery Act passed by Congress and signed by the President in May 2009. The Commission's statutory instructions were to investigate the following 22 specific topics relating to the collapse of major financial institutions that failed or would have failed if not for exceptional government assistance:[91]

- Fraud and abuse in the financial sector, including fraud and abuse toward consumers in the mortgage sector;
- Federal and State financial regulators, including the extent to which they enforced, or failed to enforce statutory, regulatory, or supervisory requirements;
- The global imbalance of savings, international capital flows, and fiscal imbalances of various governments;
- Monetary policy and the availability and terms of credit;
- Accounting practices, including, mark-to-market and fair value rules, and treatment of off-balance sheet vehicles;
- Tax treatment of financial products and investments;
- Capital requirements and regulations on leverage and liquidity, including the capital structures of regulated and non-regulated financial entities;
- Credit rating agencies in the financial system, including, reliance on credit ratings by financial institutions and Federal financial regulators, the use of credit ratings in financial regulation, and the use of credit ratings in the securitization markets;
- Lending practices and securitization, including the originate-to-distribute model for extending credit and transferring risk;
- Affiliations between insured depository institutions and securities, insurance, and other types of nonbanking companies;
- The concept that certain institutions are 'too-big-to-fail' and its impact on market expectations;
- Corporate governance, including the impact of company conversions from partnerships to corporations;
- Compensation structures;
- Changes in compensation for employees of financial companies, as compared to compensation for others with similar skill sets in the labor market;

[91] http://cybercemetery.unt.edu/archive/fcic/20110310173855/http://www.fcic.gov/about/history

- The legal and regulatory structure of the United States housing market;
- Derivatives and unregulated financial products and practices, including credit default swaps;
- Short-selling;
- Financial institution reliance on numerical models, including risk models and credit ratings;
- The legal and regulatory structure governing financial institutions, including the extent to which the structure creates the opportunity for financial institutions to engage in regulatory arbitrage;
- The legal and regulatory structure governing investor and mortgagor protection;
- Financial institutions and government-sponsored enterprises; and
- The quality of due diligence undertaken by financial institutions.

After reviewing millions of pages of documents and conducting over 700 interviews, the Commission delivered its report on 27 January 2011. The conclusions are as follows (reproduced here in full):[92]

<u>Recommended Reading: Textbook Chapters 5 and 19</u>

92 http://cybercemetery.unt.edu/archive/fcic/20110310173539/http://c0182732.cdn1.cloudfiles.rackspacecloud.com/fcic_final_report_conclusions.pdf

CONCLUSIONS OF THE

FINANCIAL CRISIS INQUIRY COMMISSION

The Financial Crisis Inquiry Commission has been called upon to examine the financial and economic crisis that has gripped our country and explain its causes to the American people. We are keenly aware of the significance of our charge, given the economic damage that America has suffered in the wake of the greatest financial crisis since the Great Depression.

Our task was first to determine what happened and how it happened so that we could understand why it happened. Here we present our conclusions. We encourage the American people to join us in making their own assessments based on the evidence gathered in our inquiry. If we do not learn from history, we are unlikely to fully recover from it. Some on Wall Street and in Washington with a stake in the status quo may be tempted to wipe from memory the events of this crisis, or to suggest that no one could have foreseen or prevented them. This report endeavors to expose the facts, identify responsibility, unravel myths, and help us understand how the crisis could have been avoided. It is an attempt to record history, not to rewrite it, nor allow it to be rewritten.

The subject of this report is of no small consequence to this nation. The profound events of 2007 and 2008 were neither bumps in the road nor an accentuated dip in the financial and business cycles we have come to expect in a free market economic system. This was a fundamental disruption—a financial upheaval, if you will—that wreaked havoc in communities and neighbourhoods across this country. As this report goes to print, there are more than 26 million Americans who are out of work, cannot find full-time work, or have given up looking for work. About four million families have lost their homes to foreclosure and another four and a half million have slipped into the foreclosure process or are seriously behind on their mortgage payments. Nearly $11 trillion in household wealth has vanished, with retirement accounts and life savings swept away. Businesses, large and small, have felt the sting of a deep recession. There is much anger about what has transpired, and justifiably so. Many people who abided by all the rules now find themselves out of work and uncertain about their future prospects. The collateral damage of this crisis has been real people and real communities. The impacts of this crisis are likely to be felt for a generation. And the nation faces no easy path to renewed economic strength.

Like so many Americans, we began our exploration with our own views and some preliminary knowledge about how the world's strongest financial system came to the brink of collapse. Even at the time of our appointment to this independent panel, much had already been written and said about the crisis. Yet all of us have been deeply affected by what we have learned in the course of our inquiry. We have been at various times fascinated, surprised, and even shocked by what we saw, heard, and read. Ours has been a journey of revelation.

Much attention over the past two years has been focused on the decisions by the federal government to provide massive financial assistance to stabilize the financial system and rescue large financial institutions that were deemed too systemically important to fail. Those decisions—and the deep emotions surrounding them—will be debated long into the future. But our mission was to ask and answer this central question: how did it come to pass that in 2008 our nation was forced to choose between two stark and painful alternatives—either risk the total collapse of our financial system and economy or inject trillions of taxpayer dollars into the financial system and an array of companies, as millions of Americans still lost their jobs, their savings, and their homes?

In this report, we detail the events of the crisis. But a simple summary, as we see it, is useful at the outset. While the vulnerabilities that created the potential for crisis were years in the making, it was the collapse of the housing bubble—fueled by low interest rates, easy and available credit, scant regulation, and toxic mortgages—that was the spark that ignited a string of events, which led to a full-blown crisis in the fall of 2008. Trillions of dollars in risky mortgages had become embedded throughout the financial system, as mortgage-related securities were packaged, repackaged, and sold to investors around the world. When the bubble burst, hundreds of billions of dollars in losses in mortgages and mortgage-related securities shook markets as well as financial institutions that had significant exposures to those mortgages and had borrowed heavily against them. This happened not just in the United States but around the world. The losses were magnified by derivatives such as synthetic securities.

The crisis reached seismic proportions in September 2008 with the failure of Lehman Brothers and the impending collapse of the insurance giant American International Group (AIG). Panic fanned by a lack of transparency of the balance sheets of major financial institutions, coupled with a tangle of interconnections among institutions perceived to be "too big to fail," caused the credit markets to seize up. Trading ground to a halt. The stock market plummeted. The economy plunged into a deep recession. The financial system we examined bears little resemblance to that of our parents' generation. The changes in the past three decades alone have been remarkable. The financial markets have become increasingly globalized. Technology has transformed the efficiency, speed, and complexity of financial instruments and transactions. There is broader access to and lower costs of financing than ever before. And the financial sector itself has become a much more dominant force in our economy.

From 1978 to 2007, the amount of debt held by the financial sector soared from $3 trillion to $36 trillion, more than doubling as a share of gross domestic product. The very nature of many Wall Street firms changed—from relatively staid private partnerships to publicly traded corporations taking greater and more diverse kinds of risks. By 2005, the 10 largest U.S. commercial banks held 55% of the industry's assets, more than double the level held in 1990. On the eve of the crisis in 2006, financial sector profits constituted 27% of all corporate profits in the United States, up from 15% in 1980. Understanding this transformation has been critical to the Commission's analysis.

Now to our major findings and conclusions, which are based on the facts contained in this report: they are offered with the hope that lessons may be learned to help avoid future catastrophe.

• **We conclude this financial crisis was avoidable.** The crisis was the result of human action and inaction, not of Mother Nature or computer models gone haywire. The captains of finance and the public stewards of our financial system ignored warnings and failed to question, understand, and manage evolving risks within a system essential to the well-being of the American public. Theirs was a big miss, not a stumble. While the business cycle cannot be repealed, a crisis of this magnitude need not have occurred. To paraphrase Shakespeare, the fault lies not in the stars, but in us.

Despite the expressed view of many on Wall Street and in Washington that the crisis could not have been foreseen or avoided, there were warning signs. The tragedy was that they were ignored or discounted. There was an explosion in risky subprime lending and securitization, an unsustainable rise in housing prices, widespread reports of egregious and predatory lending practices, dramatic increases in household mortgage debt, and exponential growth in financial firms' trading activities, unregulated derivatives, and short-term "repo" lending markets, among many other red flags. Yet there was pervasive permissiveness; little meaningful action was taken to quell the threats in a timely manner.

The prime example is the Federal Reserve's pivotal failure to stem the flow of toxic mortgages, which it could have done by setting prudent mortgage-lending standards. The Federal Reserve was the one entity empowered to do so and it did not. The record of our examination is replete with evidence of other failures: financial institutions made, bought, and sold mortgage securities they never examined, did not care to examine, or knew to be defective; firms depended on tens of billions of dollars of borrowing that had to be renewed each and every night, secured by subprime mortgage securities; and major firms and investors blindly relied on credit rating agencies as their arbiters of risk. What else could one expect on a highway where there were neither speed limits nor neatly painted lines?

• **We conclude widespread failures in financial regulation and supervision proved devastating to the stability of the nation's financial markets.** The sentries were not at their posts, in no small part due to the widely accepted faith in the self-correcting nature of the markets and the ability of financial institutions to effectively police themselves. More than 30 years of deregulation and reliance on self-regulation by financial institutions, championed by former Federal Reserve chairman Alan Greenspan and others, supported by successive administrations and Congresses, and actively pushed by the powerful financial industry at every turn, had stripped away key safeguards, which could have helped avoid catastrophe. This approach had opened up gaps in oversight of critical areas with trillions of dollars at risk, such as the shadow banking system and over-the-counter derivatives markets. In addition, the government permitted financial firms to pick their preferred regulators in what became a race to the weakest supervisor.

Yet we do not accept the view that regulators lacked the power to protect the financial system. They had ample power in many arenas and they chose not to use it. To give just three examples: the Securities and Exchange Commission could have required more capital and halted risky practices at the big investment banks. It did not. The Federal Reserve Bank of New York and other regulators could have clamped down on Citigroup's excesses in the run-up to the crisis. They did not. Policy makers and regulators could have stopped the runaway mortgage securitization train. They did not. In case after case after case, regulators continued to rate the institutions they oversaw as safe and sound even in the face of mounting troubles, often downgrading them just before their collapse. And where regulators lacked authority, they could have sought it. Too often, they lacked the political will—in a political and ideological environment that constrained it—as well as the fortitude to critically challenge the institutions and the entire system they were entrusted to oversee.

Changes in the regulatory system occurred in many instances as financial markets evolved. But as the report will show, the financial industry itself played a key role in weakening regulatory constraints on institutions, markets, and products. It did not surprise the Commission that an industry of such wealth and power would exert pressure on policy makers and regulators. From 1999 to 2008, the financial sector expended $2.7 billion in reported federal lobbying expenses; individuals and political action committees in the sector made more than $1 billion in campaign contributions. What troubled us was the extent to which the nation was deprived of the necessary strength and independence of the oversight necessary to safeguard financial stability.

• **We conclude dramatic failures of corporate governance and risk management at many systemically important financial institutions were a key cause of this crisis.** There was a view that instincts for self-preservation inside major financial firms would shield them from fatal risk-taking without the need for a steady regulatory hand, which, the firms argued, would stifle innovation. Too many of these institutions acted recklessly, taking on too much risk, with too little capital, and with too much dependence on short-term funding. In many respects, this reflected a fundamental change in these institutions, particularly the large investment banks and bank holding companies, which focused their activities increasingly on risky trading activities that produced hefty profits. They took on enormous exposures in acquiring and supporting subprime lenders and creating, packaging, repackaging, and selling trillions of dollars in mortgage-related securities, including synthetic financial products. Like Icarus, they never feared flying ever closer to the sun.

Many of these institutions grew aggressively through poorly executed acquisition and integration strategies that made effective management more challenging. The CEO of Citigroup told the Commission that a $40 billion position in highly rated mortgage securities would "not in any way have excited my attention," and the cohead of Citigroup's investment bank said he spent "a small fraction of 1%" of his time on those securities. In this instance, too big to fail meant too big to manage.

Financial institutions and credit rating agencies embraced mathematical models as reliable predictors of risks, replacing judgment in too many instances. Too often, risk management became risk justification.

Compensation systems—designed in an environment of cheap money, intense competition, and light regulation—too often rewarded the quick deal, the short-term gain—without proper consideration of long-term consequences. Often, those systems encouraged the big bet—where the payoff on the upside could be huge and the downside limited. This was the case up and down the line—from the corporate boardroom to the mortgage broker on the street.

Our examination revealed stunning instances of governance breakdowns and irresponsibility. You will read, among other things, about AIG senior management's ignorance of the terms and risks of the company's $79 billion derivatives exposure to mortgage-related securities; Fannie Mae's quest for bigger market share, profits, and bonuses, which led it to ramp up its exposure to risky loans and securities as the housing market was peaking; and the costly surprise when Merrill Lynch's top management realized that the company held $55 billion in "super-senior" and supposedly "super-safe" mortgage-related securities that resulted in billions of dollars in losses.

• **We conclude a combination of excessive borrowing, risky investments, and lack of transparency put the financial system on a collision course with crisis.** Clearly, this vulnerability was related to failures of corporate governance and regulation, but it is significant enough by itself to warrant our attention here.

In the years leading up to the crisis, too many financial institutions, as well as too many households, borrowed to the hilt, leaving them vulnerable to financial distress or ruin if the value of their investments declined even modestly. For example, as of 2007, the five major investment banks—Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, and Morgan Stanley—were operating with extraordinarily thin capital. By one measure, their leverage ratios were as high as 40 to 1, meaning for every $40 in assets, there was only $1 in capital to cover losses. Less than a 3% drop in asset values could wipe out a firm. To make matters worse, much of their borrowing was short-term, in the overnight market—meaning the borrowing had to be renewed each and every day. For example, at the end of 2007, Bear Stearns had $11.8 billion in equity and $383.6 billion in liabilities and was borrowing as much as $70 billion in the overnight market. It was the equivalent of a small business with $50,000 in equity borrowing $1.6 million, with $296,750 of that due each and every day. One can't really ask "What were they thinking?" when it seems that too many of them were thinking alike.

And the leverage was often hidden—in derivatives positions, in off-balance-sheet entities, and through "window dressing" of financial reports available to the investing public.

The kings of leverage were Fannie Mae and Freddie Mac, the two behemoth government-sponsored enterprises (GSEs). For example, by the end of 2007, Fannie's and Freddie's combined leverage ratio, including loans they owned and guaranteed, stood at 75 to 1.

But financial firms were not alone in the borrowing spree: from 2001 to 2007, national mortgage debt almost doubled, and the amount of mortgage debt per household rose more than 63% from $91,500 to $149,500, even while wages were essentially stagnant. When the housing downturn hit, heavily indebted financial firms and families alike were walloped.

The heavy debt taken on by some financial institutions was exacerbated by the risky assets they were acquiring with that debt. As the mortgage and real estate markets churned out riskier and riskier loans and securities, many financial institutions loaded up on them. By the end of 2007, Lehman had amassed $111 billion in commercial and residential real estate holdings and securities, which was almost twice what it held just two years before, and more than four times its total equity. And again, the risk wasn't being taken on just by the big financial firms, but by families, too. Nearly one in 10 mortgage borrowers in 2005 and 2006 took out "option ARM" loans, which meant they could choose to make payments so low that their mortgage balances rose every month.

Within the financial system, the dangers of this debt were magnified because transparency was not required or desired. Massive, short-term borrowing, combined with obligations unseen by others in the market, heightened the chances the system could rapidly unravel. In the early part of the 20th century, we erected a series of protections—the Federal Reserve as a lender of last resort, federal deposit insurance, ample regulations—to provide a bulwark against the panics that had regularly plagued America's banking system in the 19th century.

Yet, over the past 30-plus years, we permitted the growth of a shadow banking system—opaque and laden with shortterm debt—that rivaled the size of the traditional banking system. Key components of the market—for example, the multitrillion-dollar repo lending market, off-balance-sheet entities, and the use of over-the-counter derivatives—were hidden from view, without the protections we had constructed to prevent financial meltdowns. We had a 21st-century financial system with 19th-century safeguards.

When the housing and mortgage markets cratered, the lack of transparency, the extraordinary debt loads, the short-term loans, and the risky assets all came home to roost. What resulted was panic. We had reaped what we had sown.

• **We conclude the government was ill prepared for the crisis, and its inconsistent response added to the uncertainty and panic in the financial markets.** As part of our charge, it was appropriate to review government actions taken in response to the developing crisis, not just those policies or actions that preceded it, to determine if any of those responses contributed to or exacerbated the crisis.

As our report shows, key policy makers—the Treasury Department, the Federal Reserve Board, and the Federal Reserve Bank of New York—who were best positioned to watch over our markets were ill prepared for the events of 2007 and 2008. Other agencies were also behind the curve. They were hampered because they did not have a clear grasp of the financial system they were charged with overseeing, particularly as it had evolved in the years leading up to the crisis. This was in no small measure due to the lack of transparency in key markets. They thought risk had been diversified when, in fact, it had been concentrated. Time and again, from the spring of 2007 on, policy makers and regulators were caught off guard as the contagion spread, responding on an ad hoc basis with specific programs to put fingers in the dike. There was no comprehensive and strategic plan for containment, because they lacked a full understanding of the risks and interconnections in the financial markets. Some regulators have conceded this error. We had allowed the system to race ahead of our ability to protect it.

While there was some awareness of, or at least a debate about, the housing bubble, the record reflects that senior public officials did not recognize that a bursting of the bubble could threaten the entire financial system. Throughout the summer of 2007, both Federal Reserve Chairman Ben Bernanke and Treasury Secretary Henry Paulson offered public assurances that the turmoil in the subprime mortgage markets would be contained. When Bear Stearns's hedge funds, which were heavily invested in mortgage-related securities, imploded in June 2007, the Federal Reserve discussed the implications of the collapse. Despite the fact that so many other funds were exposed to the same risks as those hedge funds, the Bear Stearns funds were thought to be "relatively unique." Days before the collapse of Bear Stearns in March 2008, SEC Chairman Christopher Cox expressed "comfort about the capital cushions" at the big investment banks. It was not until August 2008, just weeks before the government takeover of Fannie Mae and Freddie Mac, that the Treasury Department understood the full measure of the dire financial conditions of those two institutions.

And just a month before Lehman's collapse, the Federal Reserve Bank of New York was still seeking information on the exposures created by Lehman's more than 900,000 derivatives contracts.

In addition, the government's inconsistent handling of major financial institutions during the crisis—the decision to rescue Bear Stearns and then to place Fannie Mae and Freddie Mac into conservatorship, followed by its decision not to save Lehman Brothers and then to save AIG—increased uncertainty and panic in the market.

In making these observations, we deeply respect and appreciate the efforts made by Secretary Paulson, Chairman Bernanke, and Timothy Geithner, formerly president of the Federal Reserve Bank of New York and now treasury secretary, and so many others who labored to stabilize our financial system and our economy in the most chaotic and challenging of circumstances.

• **We conclude there was a systemic breakdown in accountability and ethics. The integrity of our financial markets and the public's trust in those markets are essential to the economic well-being of our nation.** The soundness and the sustained prosperity of the financial system and our economy rely on the notions of fair dealing, responsibility, and transparency. In our economy, we expect businesses and individuals to pursue profits, at the same time that they produce products and services of quality and conduct themselves well.

Unfortunately—as has been the case in past speculative booms and busts—we witnessed an erosion of standards of responsibility and ethics that exacerbated the financial crisis. This was not universal, but these breaches stretched from the ground level to the corporate suites. They resulted not only in significant financial consequences but also in damage to the trust of investors, businesses, and the public in the financial system.

For example, our examination found, according to one measure, that the percentage of borrowers who defaulted on their mortgages within just a matter of months after taking a loan nearly doubled from the summer of 2006 to late 2007. This data indicates they likely took out mortgages that they never had the capacity or intention to pay. You will read about mortgage brokers who were paid "yield spread premiums" by lenders to put borrowers into higher-cost loans so they would get bigger fees, often never disclosed to borrowers. The report catalogues the rising incidence of mortgage fraud, which flourished in an environment of collapsing lending standards and lax regulation. The number of suspicious activity reports—reports of possible financial crimes filed by depository banks and their affiliates—related to mortgage fraud grew 20-fold between 1996 and 2005 and then more than doubled again between 2005 and 2009. One study places the losses resulting from fraud on mortgage loans made between 2005 and 2007 at $112 billion.

Lenders made loans that they knew borrowers could not afford and that could cause massive losses to investors in mortgage securities. As early as September 2004, countrywide executives recognized that many of the loans they were originating could result in "catastrophic consequences." Less than a year later, they noted that certain high-risk loans they were making could result not only in foreclosures but also in "financial and reputational catastrophe" for the firm. But they did not stop. And the report documents that major financial institutions ineffectively sampled loans they were purchasing to package and sell to investors. They knew a significant percentage of the sampled loans did not meet their own underwriting standards or those of the originators. Nonetheless, they sold those securities to investors. The Commission's review of many prospectuses provided to investors found that this critical information was not disclosed.

THESE CONCLUSIONS must be viewed in the context of human nature and individual and societal responsibility. First, to pin this crisis on mortal flaws like greed and hubris would be simplistic. It was the failure to account for human weakness that is relevant to this crisis.

Second, we clearly believe the crisis was a result of human mistakes, misjudgments, and misdeeds that resulted in systemic failures for which our nation has paid dearly. As you read this report, you will see that specific firms and individuals acted irresponsibly. Yet a crisis of this magnitude cannot be the work of a few bad actors, and such was not the case here. At the same time, the breadth of this crisis does not mean that "everyone is at fault"; many firms and individuals did not participate in the excesses that spawned disaster.

We do place special responsibility with the public leaders charged with protecting our financial system, those entrusted to run our regulatory agencies, and the chief executives of companies whose failures drove us to crisis. These individuals sought and accepted positions of significant responsibility and obligation. Tone at the top does matter and, in this instance, we were let down. No one said "no."

But as a nation, we must also accept responsibility for what we permitted to occur. Collectively, but certainly not unanimously, we acquiesced to or embraced a system, a set of policies and actions, that gave rise to our present predicament.

* * *

THIS REPORT DESCRIBES THE EVENTS and the system that propelled our nation toward crisis. The complex machinery of our financial markets has many essential gears—some of which played a critical role as the crisis developed and deepened. Here we render our conclusions about specific components of the system that we believe contributed significantly to the financial meltdown.

• **We conclude collapsing mortgage-lending standards and the mortgage securitization pipeline lit and spread the flame of contagion and crisis.** When housing prices fell and mortgage borrowers defaulted, the lights began to dim on Wall Street. This report catalogues the corrosion of mortgage-lending standards and the securitization pipeline that transported toxic mortgages from neighborhoods across America to investors around the globe.

Many mortgage lenders set the bar so low that lenders simply took eager borrowers' qualifications on faith, often with a willful disregard for a borrower's ability to pay. Nearly one-quarter of all mortgages made in the first half of 2005 were interestonly loans. During the same year, 68% of "option ARM" loans originated by Countrywide and Washington Mutual had low- or no-documentation requirements.

These trends were not secret. As irresponsible lending, including predatory and fraudulent practices, became more prevalent, the Federal Reserve and other regulators and authorities heard warnings from many quarters. Yet the Federal Reserve neglected its mission "to ensure the safety and soundness of the nation's banking and financial system and to protect the credit rights of consumers." It failed to build the retaining wall before it was too late. And the Office of the Comptroller of the Currency and the Office of Thrift Supervision, caught up in turf wars, preempted state regulators from reining in abuses.

While many of these mortgages were kept on banks' books, the bigger money came from global investors who clamored to put their cash into newly created mortgage-related securities. It appeared to financial institutions, investors, and regulators alike that risk had been conquered: the investors held highly rated securities they thought were sure to perform; the banks thought they had taken the riskiest loans off their books; and regulators saw firms making profits and borrowing costs reduced. But each step in the mortgage securitization pipeline depended on the next step to keep demand going. From the speculators who flipped houses to the mortgage brokers who scouted the loans, to the lenders who issued the mortgages, to the financial firms that created the mortgage-backed securities, collateralized debt obligations (CDOs), CDOs squared, and synthetic CDOs: no one in this pipeline of toxic mortgages had enough skin in the game. They all believed they could off-load their risks on a moment's notice to the next person in line. They were wrong. When borrowers stopped making mortgage payments, the losses—amplified by derivatives—rushed through the pipeline. As it turned out, these losses were concentrated in a set of systemically important financial institutions.

In the end, the system that created millions of mortgages so efficiently has proven to be difficult to unwind. Its complexity has erected barriers to modifying mortgages so families can stay in their homes and has created further uncertainty about the health of the housing market and financial institutions.

• **We conclude over-the-counter derivatives contributed significantly to this crisis.** The enactment of legislation in 2000 to ban the regulation by both the federal and state governments of over-the-counter (OTC) derivatives was a key turning point in the march toward the financial crisis.

From financial firms to corporations, to farmers, and to investors, derivatives have been used to hedge against, or speculate on, changes in prices, rates, or indices or even on events such as the potential defaults on debts. Yet, without any oversight, OTC derivatives rapidly spiraled out of control and out of sight, growing to $673 trillion in notional amount. This report explains the uncontrolled leverage; lack of transparency, capital, and collateral requirements; speculation; interconnections among firms; and concentrations of risk in this market.

OTC derivatives contributed to the crisis in three significant ways. First, one type of derivative—credit default swaps (CDS)—fueled the mortgage securitization pipeline. CDS were sold to investors to protect against the default or decline in value of mortgage-related securities backed by risky loans. Companies sold protection—to the tune of $79 billion, in AIG's case—to investors in these newfangled mortgage securities, helping to launch and expand the market and, in turn, to further fuel the housing bubble.

Second, CDS were essential to the creation of synthetic CDOs. These synthetic CDOs were merely bets on the performance of real mortgage-related securities. They amplified the losses from the collapse of the housing bubble by allowing multiple bets on the same securities and helped spread them throughout the financial system. Goldman Sachs alone packaged and sold $73 billion in synthetic CDOs from July 1, 2004, to May 31, 2007. Synthetic CDOs created by Goldman referenced more than 3,400 mortgage securities, and 610 of them were referenced at least twice. This is apart from how many times these securities may have been referenced in synthetic CDOs created by other firms.

Finally, when the housing bubble popped and crisis followed, derivatives were in the center of the storm. AIG, which had not been required to put aside capital reserves as a cushion for the protection it was selling, was bailed out when it could not meet its obligations. The government ultimately committed more than $180 billion because of concerns that AIG's collapse would trigger cascading losses throughout the global financial system. In addition, the existence of millions of derivatives contracts of all types between systemically important financial institutions—unseen and unknown in this unregulated market—added to uncertainty and escalated panic, helping to precipitate government assistance to those institutions.

• **We conclude the failures of credit rating agencies were essential cogs in the wheel of financial destruction. The three credit rating agencies were key enablers of the financial meltdown.** The mortgage-related securities at the heart of the crisis could not have been marketed and sold without their seal of approval. Investors relied on them, often blindly. In some cases, they were obligated to use them, or regulatory capital standards were hinged on them. This crisis could not have happened without the rating agencies. Their ratings helped the market soar and their downgrades through 2007 and 2008 wreaked havoc across markets and firms.

In our report, you will read about the breakdowns at Moody's, examined by the Commission as a case study. From 2000 to 2007, Moody's rated nearly 45,000 mortgage-related securities as triple-A. This compares with six private-sector companies in the United States that carried this coveted rating in early 2010. In 2006 alone, Moody's put its triple-A stamp of approval on 30 mortgage-related securities every working day. The results were disastrous: 83% of the mortgage securities rated triple-A that year ultimately were downgraded.

You will also read about the forces at work behind the breakdowns at Moody's, including the flawed computer models, the pressure from financial firms that paid for the ratings, the relentless drive for market share, the lack of resources to do the job despite record profits, and the absence of meaningful public oversight. And you will see that without the active participation of the rating agencies, the market for mortgage-related securities could not have been what it became.

* * *

THERE ARE MANY COMPETING VIEWS as to the causes of this crisis. In this regard, the Commission has endeavoured to address key questions posed to us. Here we discuss three: capital availability and excess liquidity, the role of Fannie Mae and Freddie Mac (the GSEs), and government housing policy.

First, as to the matter of excess liquidity: in our report, we outline monetary policies and capital flows during the years leading up to the crisis. Low interest rates, widely available capital, and international investors seeking to put their money in real estate assets in the United States were prerequisites for the creation of a credit bubble.

Those conditions created increased risks, which should have been recognized by market participants, policy makers, and regulators. However, it is the Commission's conclusion that excess liquidity did not need to cause a crisis. It was the failures outlined above—including the failure to effectively rein in excesses in the mortgage and financial markets—that were the principal causes of this crisis. Indeed, the availability of well-priced capital—both foreign and domestic—is an opportunity for economic expansion and growth if encouraged to flow in productive directions.

Second, we examined the role of the GSEs, with Fannie Mae serving as the Commission's case study in this area. These government-sponsored enterprises had a deeply flawed business model as publicly traded corporations with the implicit backing of and subsidies from the federal government and with a public mission. Their $5 trillion mortgage exposure and market position were significant. In 2005 and 2006, they decided to ramp up their purchase and guarantee of risky mortgages, just as the housing market was peaking. They used their political power for decades to ward off effective regulation and oversight—spending $164 million on lobbying from 1999 to 2008. They suffered from many of the same failures of corporate governance and risk management as the Commission discovered in other financial firms. Through the third quarter of 2010, the Treasury Department had provided $151 billion in financial support to keep them afloat.

We conclude that these two entities contributed to the crisis, but were not a primary cause. Importantly, GSE mortgage securities essentially maintained their value throughout the crisis and did not contribute to the significant financial firm losses that were central to the financial crisis.

The GSEs participated in the expansion of subprime and other risky mortgages, but they followed rather than led Wall Street and other lenders in the rush for fool's gold. They purchased the highest rated non-GSE mortgage-backed securities and their participation in this market added helium to the housing balloon, but their purchases never represented a majority of the market. Those purchases represented 10.5% of non-GSE subprime mortgage-backed securities in 2001, with the share rising to 40% in 2004, and falling back to 28% by 2008. They relaxed their underwriting standards to purchase or guarantee riskier loans and related securities in order to meet stock market analysts' and investors' expectations for growth, to regain market share, and to ensure generous compensation for their executives and employees—justifying their activities on the broad and sustained public policy support for homeownership.

The Commission also probed the performance of the loans purchased or guaranteed by Fannie and Freddie. While they generated substantial losses, delinquency rates for GSE loans were substantially lower than loans securitized by other financial firms. For example, data compiled by the Commission for a subset of borrowers with similar credit scores—scores below 660—show that by the end of 2008, GSE mortgages were far less likely to be seriously delinquent than were non-GSE securitized mortgages: 6.2% versus 28.3%.

We also studied at length how the Department of Housing and Urban Development's (HUD's) affordable housing goals for the GSEs affected their investment in risky mortgages. Based on the evidence and interviews with dozens of individuals involved in this subject area, we determined these goals only contributed marginally to Fannie's and Freddie's participation in those mortgages.

Finally, as to the matter of whether government housing policies were a primary cause of the crisis: for decades, government policy has encouraged homeownership through a set of incentives, assistance programs, and mandates. These policies were put in place and promoted by several administrations and Congresses—indeed, both Presidents Bill Clinton and George W. Bush set aggressive goals to increase homeownership.

In conducting our inquiry, we took a careful look at HUD's affordable housing goals, as noted above, and the Community Reinvestment Act (CRA). The CRA was enacted in 1977 to combat "redlining" by banks—the practice of denying credit to individuals and businesses in certain neighborhoods without regard to their creditworthiness. The CRA requires banks and savings and loans to lend, invest, and provide services to the communities from which they take deposits, consistent with bank safety and soundness.

The Commission concludes the CRA was not a significant factor in subprime lending or the crisis. Many subprime lenders were not subject to the CRA. Research indicates only 6% of high-cost loans—a proxy for subprime loans—had any connection to the law. Loans made by CRA-regulated lenders in the neighborhoods in which they were required to lend were half as likely to default as similar loans made in the same neighborhoods by independent mortgage originators not subject to the law.

Nonetheless, we make the following observation about government housing policies— they failed in this respect: As a nation, we set aggressive homeownership goals with the desire to extend credit to families previously denied access to the financial markets. Yet the government failed to ensure that the philosophy of opportunity was being matched by the practical realities on the ground. Witness again the failure of the Federal Reserve and other regulators to rein in irresponsible lending. Homeownership peaked in the spring of 2004 and then began to decline. From that point on, the talk of opportunity was tragically at odds with the reality of a financial disaster in the making.

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WHEN THIS COMMISSION began its work 18 months ago, some imagined that the events of 2008 and their consequences would be well behind us by the time we issued this report. Yet more than two years after the federal government intervened in an unprecedented manner in our financial markets, our country finds itself still grappling with the aftereffects of the calamity. Our financial system is, in many respects, still unchanged from what existed on the eve of the crisis. Indeed, in the wake of the crisis, the U.S. financial sector is now more concentrated than ever in the hands of a few large, systemically significant institutions.

While we have not been charged with making policy recommendations, the very purpose of our report has been to take stock of what happened so we can plot a new course. In our inquiry, we found dramatic breakdowns of corporate governance, profound lapses in regulatory oversight, and near fatal flaws in our financial system. We also found that a series of choices and actions led us toward a catastrophe for which we were ill prepared. These are serious matters that must be addressed and resolved to restore faith in our financial markets, to avoid the next crisis, and to rebuild a system of capital that provides the foundation for a new era of broadly shared prosperity.

The greatest tragedy would be to accept the refrain that no one could have seen this coming and thus nothing could have been done. If we accept this notion, it will happen again.

This report should not be viewed as the end of the nation's examination of this crisis. There is still much to learn, much to investigate, and much to fix.

This is our collective responsibility. It falls to us to make different choices if we want different results.